A Numerical Approach to Bonus/Malus Executive Compensation Plans

28 april 2009

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Outline

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2. Model
3. Numerical Methods
4. Results
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Introduction

Why a Bonus?

A bonus is used as a pecuniary stimulant to executives to reduce costs arising from divergent behaviour.

Current Bonus Models

- Fixed income

  Advantage: No incentive for risky behaviour.
  Disadvantage: No incentive for profit maximising behaviour.

- Vesting stocks

  Advantage: Creates an incentive to stay with the company.
  Disadvantage: Can create an incentive for stock maximising behaviour.

- Employee stock options

  Advantage: Allows to set a benchmark.
  Disadvantage:
  - Increase the volatility of the stock.
  - No difference how much someone performs below benchmark.

Controversial

Bonus that pay even when under performing are currently (very) controversial.

This is the case for *vesting stocks* and *employee stock options* with low benchmark.

Bonus-Malus Compensation Plan

A bonus scheme that is meant to motivate the executive to increase long term company value without taking too high risks.

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Use profit and loss.

Use bonus account.

Bonus-Malus Compensation Plan

Executives receive a bonus that is deposited on a bonus account. Each period executive receives a part of the bonus account. Remains are carried to the next period.

How does an Bonus-Malus compensation plan impact the behaviour of the manager?

Model

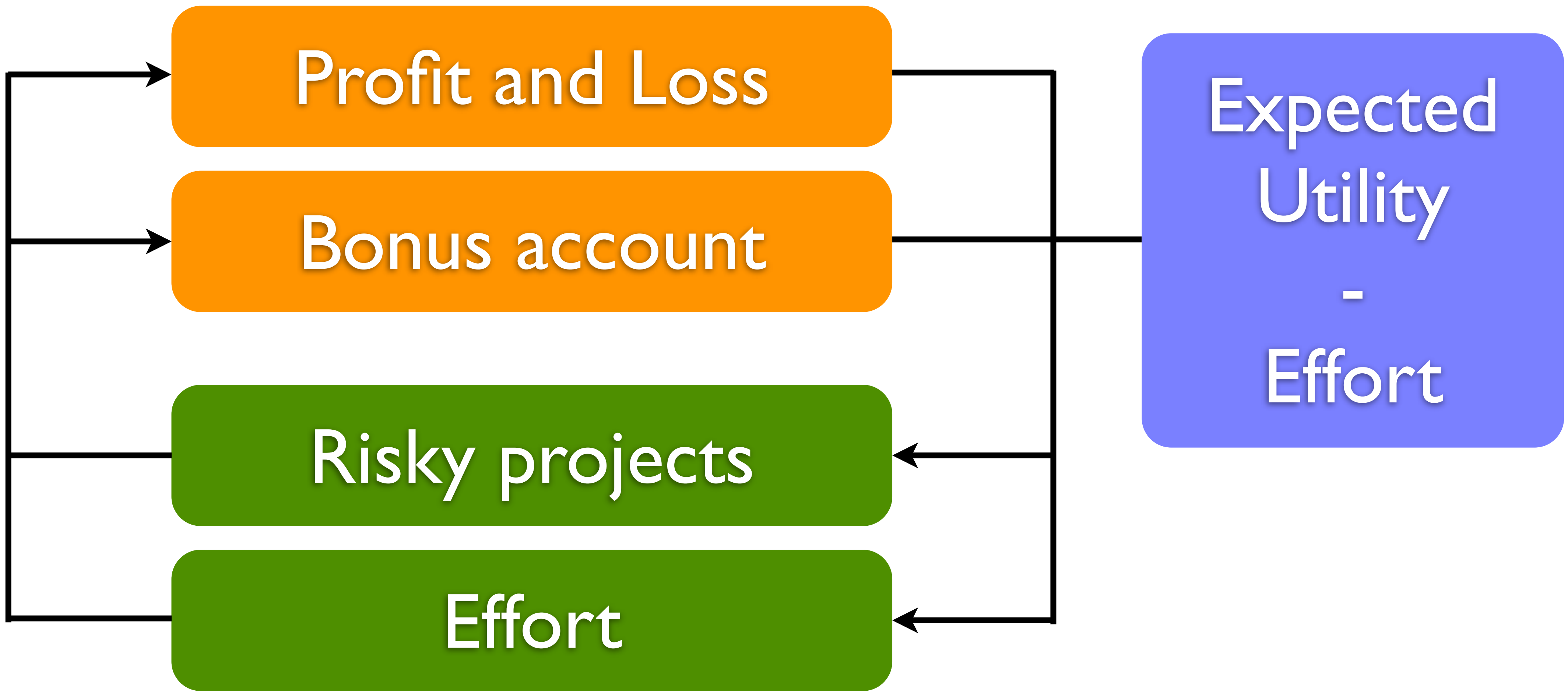
Dynamics

Executive can:

- exercise 'costly' *effort* to increase expected return on profit-and-loss account.
- accept 'free' *risky projects* to increase the volatility of the profit-and-loss account.

To maximise expected utility from payoff of the bonus account.

Formal Description



Two stochastic differential equations must be controlled to maximise expected utility from a payoff.

Stochastic Optimal Control

Optimal solution = maximum expected

utility during a infinitesimal time step

\+

optimal solution from infinitesimal time step onwards.

However

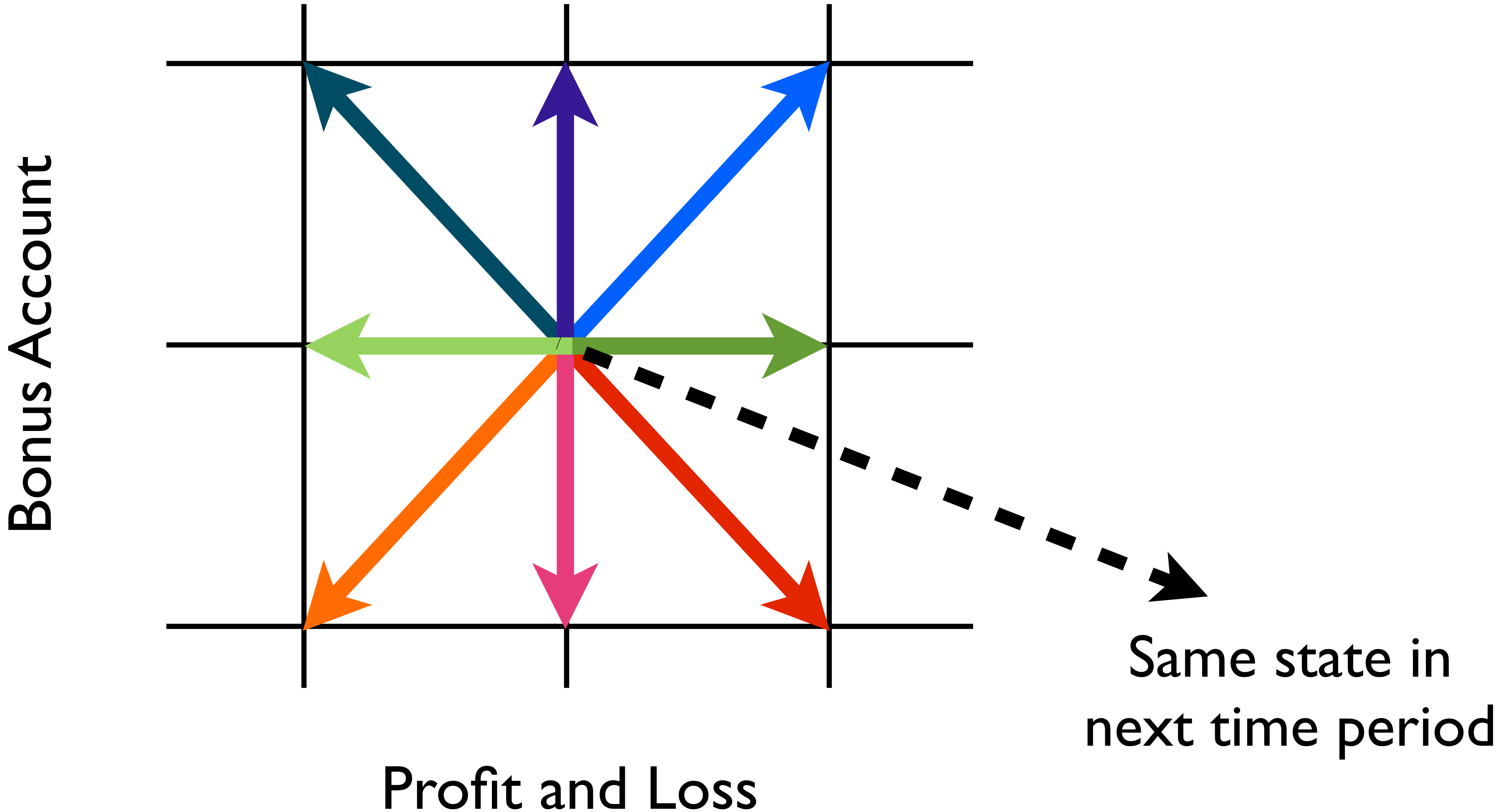
Only a limited number of explicit solutions are known. Therefore numerical methods are required.

Numerical Methods

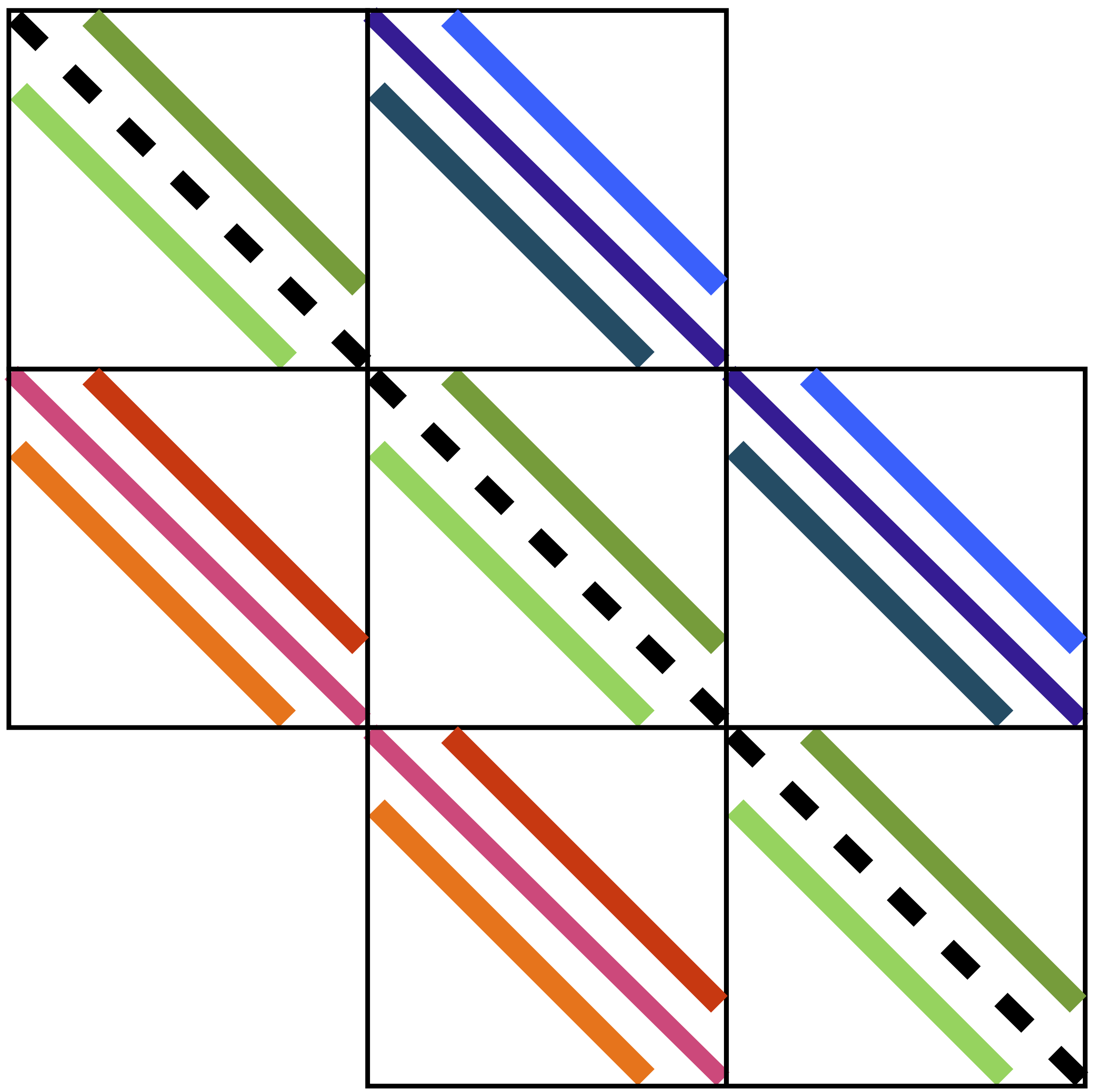
Markov Chain Approximation

1. Discretise State and Time.
2. Translate Stochastic Process to Markov Chain.
3. Solve a Dynamic Programming problem.

Transition Probabilities



Transition Probability Matrix



x Expected total utility(t) = Maximal expected utility(t + 1) + utility(t)

Stochastic Optimal Control

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Stochastic Optimal Control

Optimal solution = maximum expected

utility during a infinitesimal time step

Discretised

+

optimal solution from infinitesimal time step onwards.

Stochastic Optimal Control

Transition Matrix

Optimal solution = maximum expected

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Discretised

+

optimal solution from infinitesimal time step onwards.

Right Hand Side
system of linear equations

Stochastic Optimal Control

Solution system of linear equations

Transition Matrix

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Discretised

Right Hand Side
system of linear equations

Stochastic Optimal Control

Solution system of linear equations

Probe all possibilities in a domain

Transition Matrix

Optimal solution = maximum expected

utility during a infinitesimal time step

+

optimal solution from infinitesimal time step onwards.

Discretised

Right Hand Side
system of linear equations

Unique

Solver can approximate the solution of problems with these characteristics:

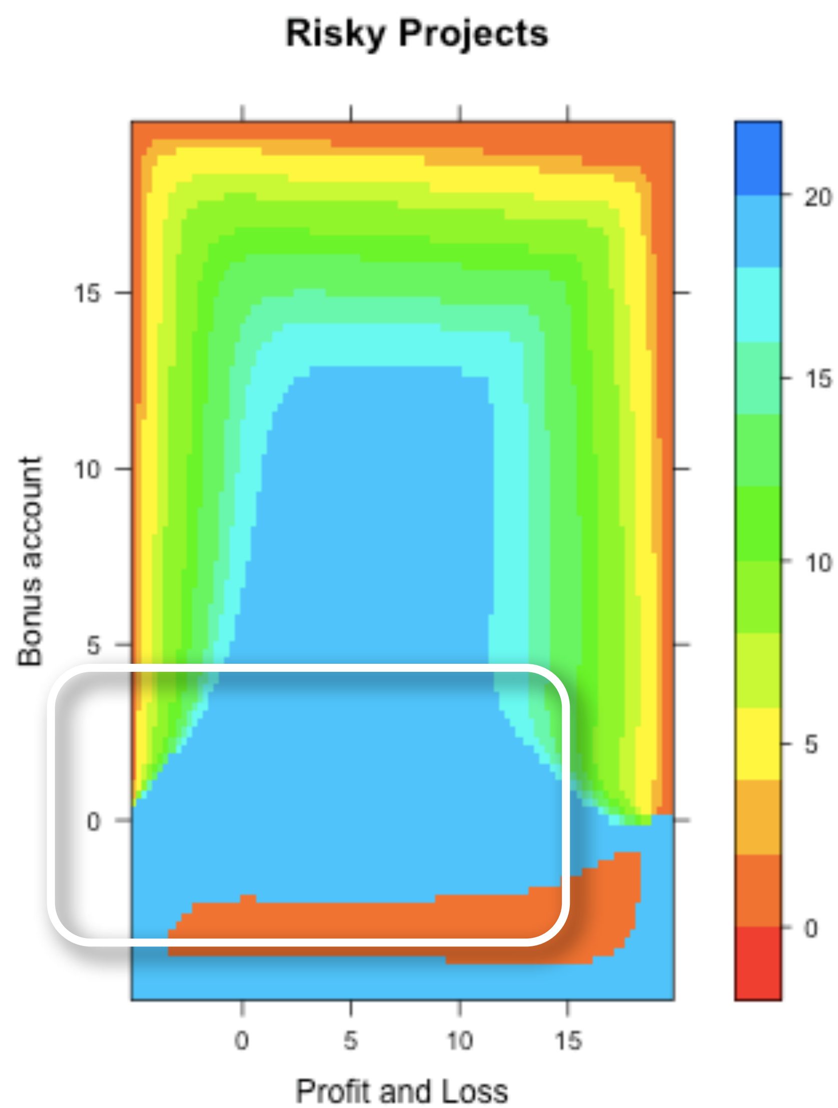
1. Finite Horizon.
2. Two Controls.
3. Two Processes (Two dimensions).

e.g. optimal hedge strategy for life insurances, hedging in incomplete markets, etc.

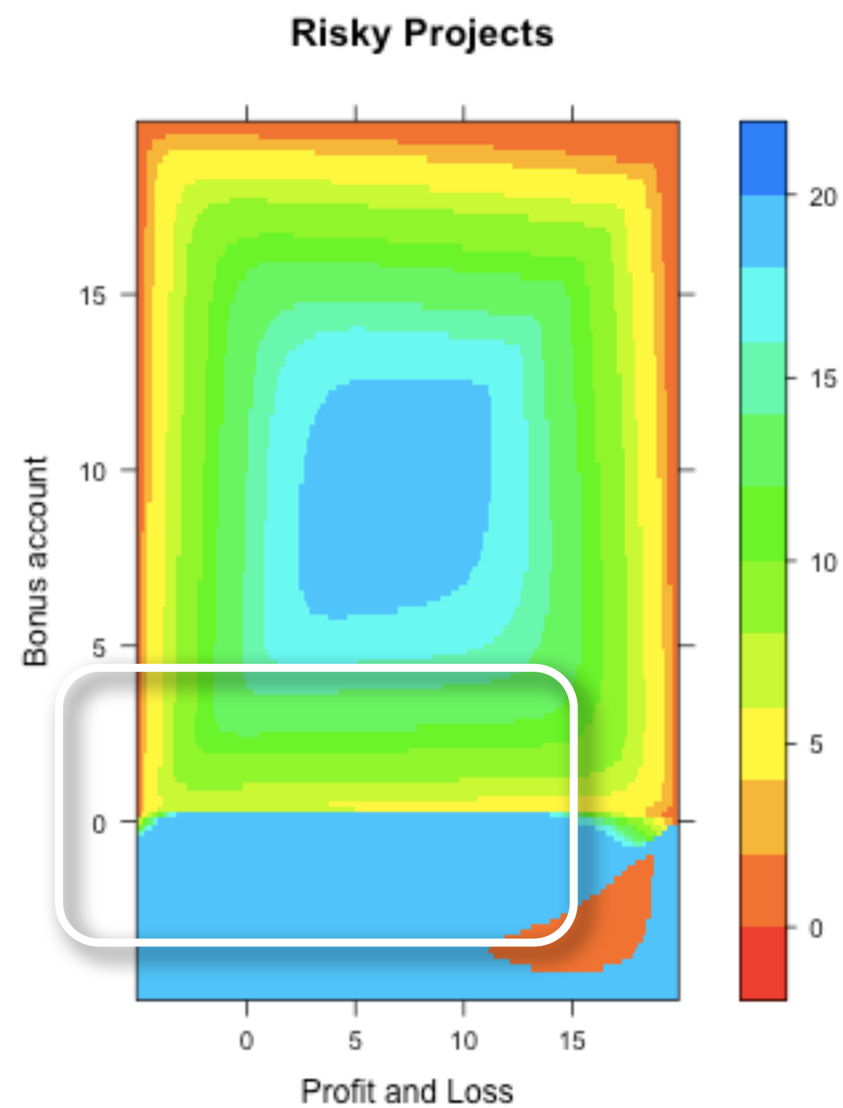
Results

(Preliminary)

Costly Effort

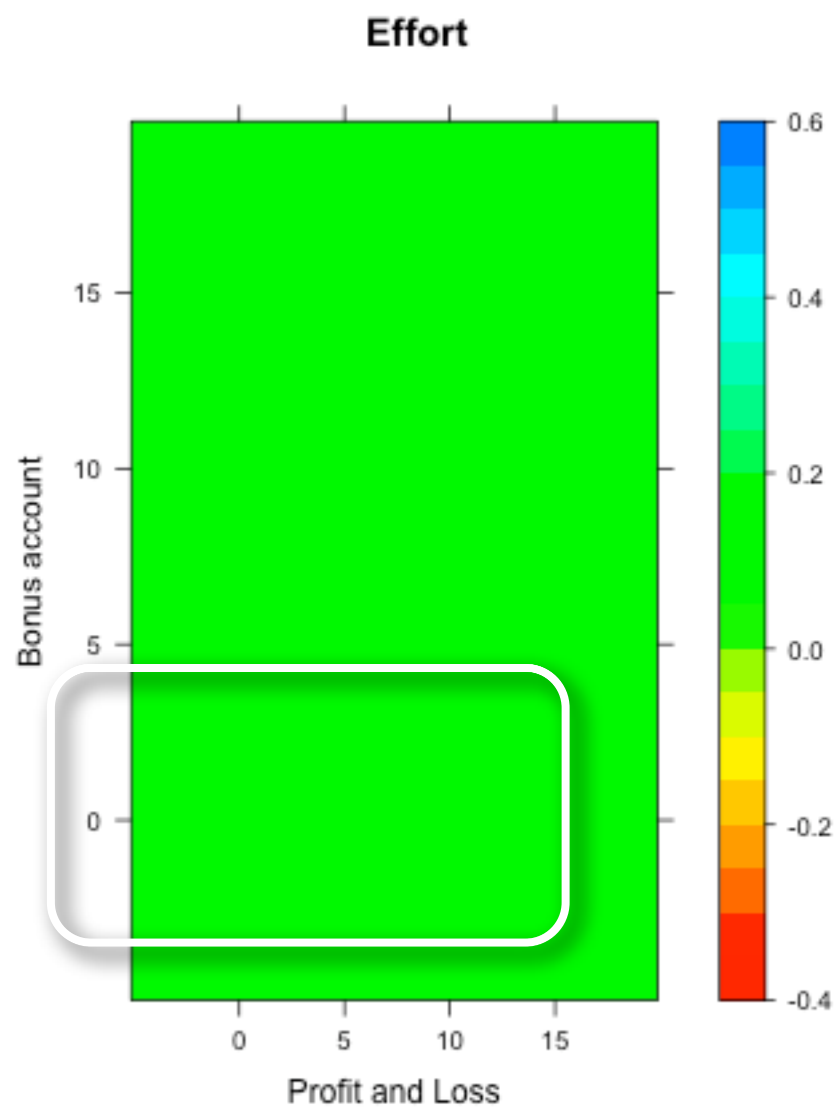


High Utility

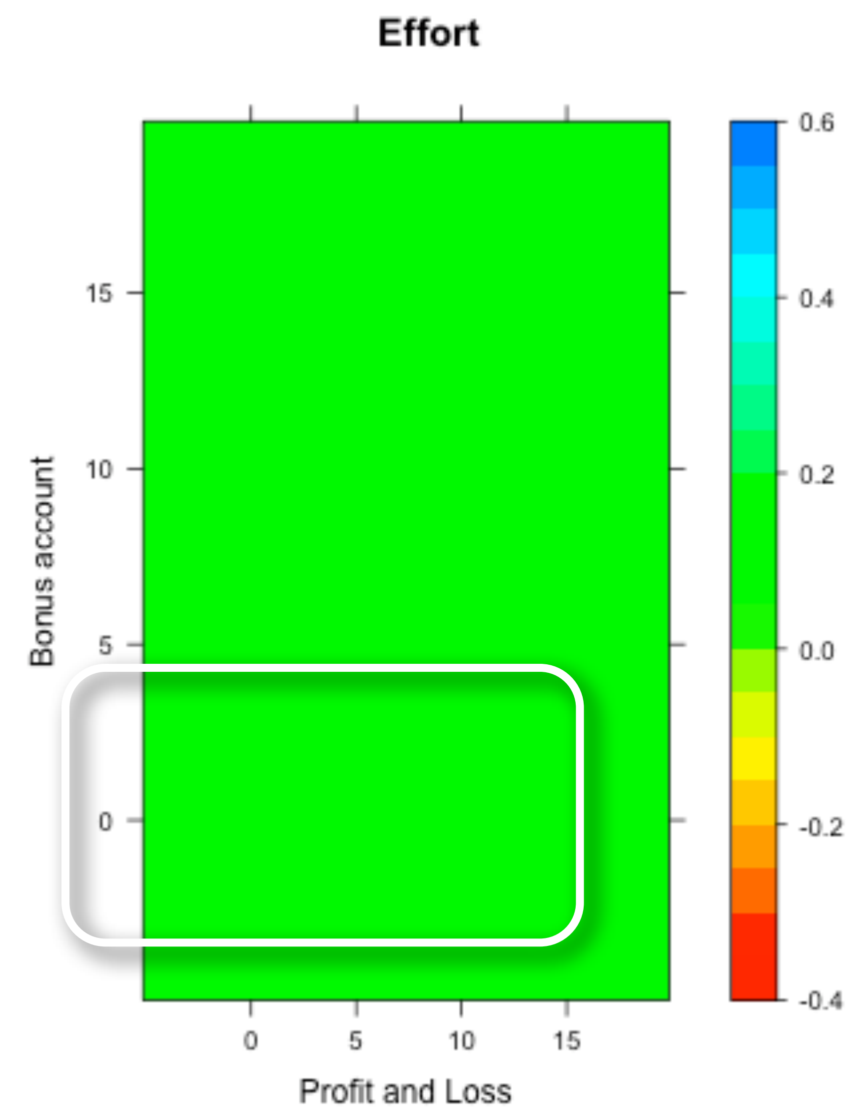


Low Utility

Costly Effort

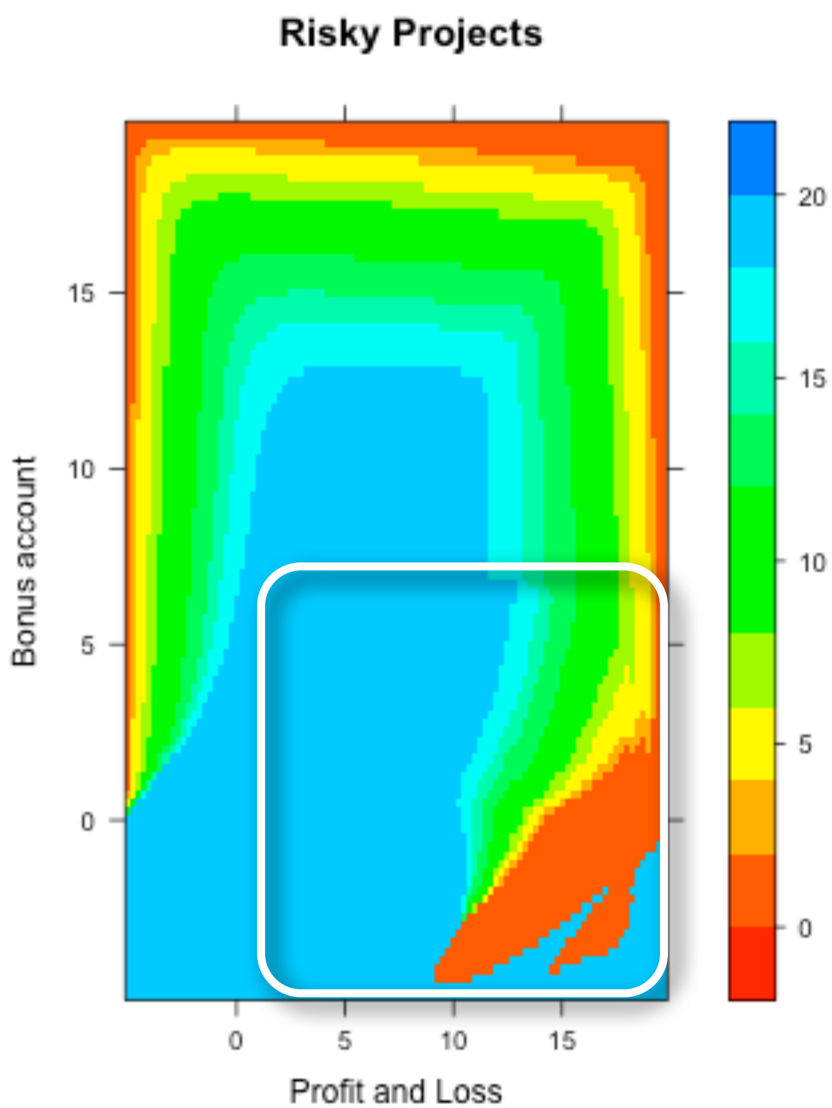


High Utility

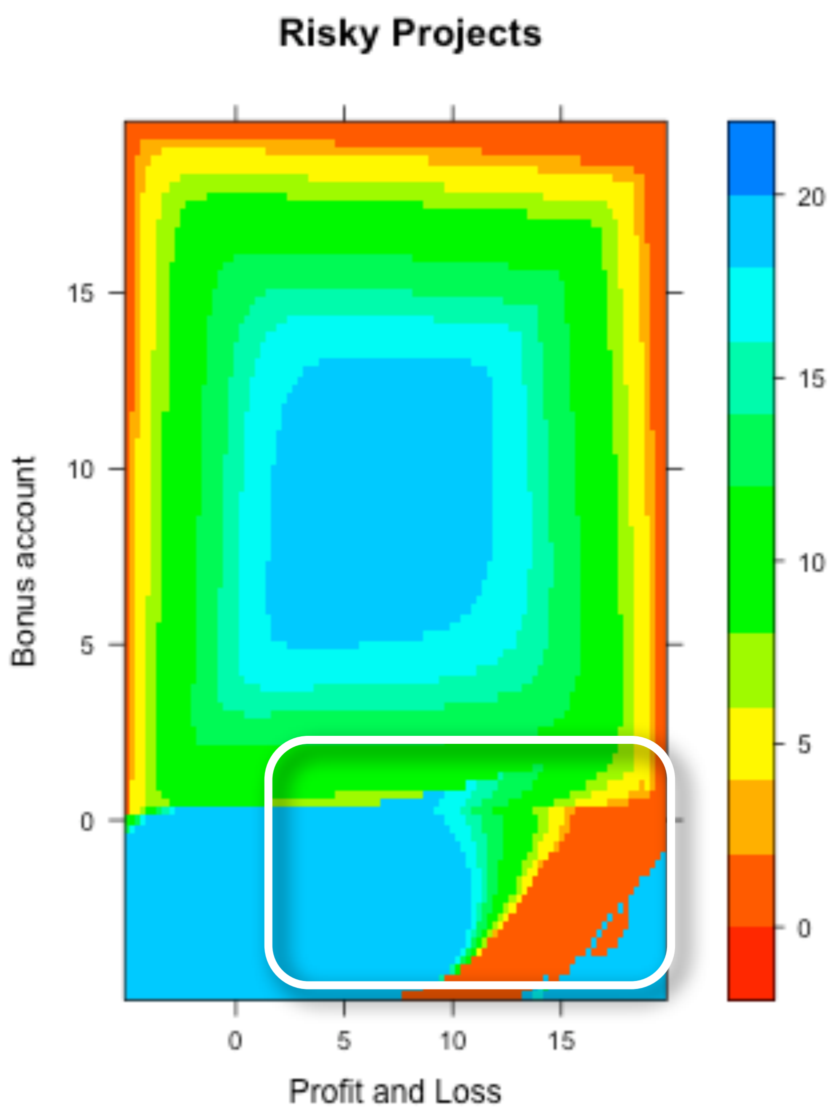


Low Utility

Cheap Effort

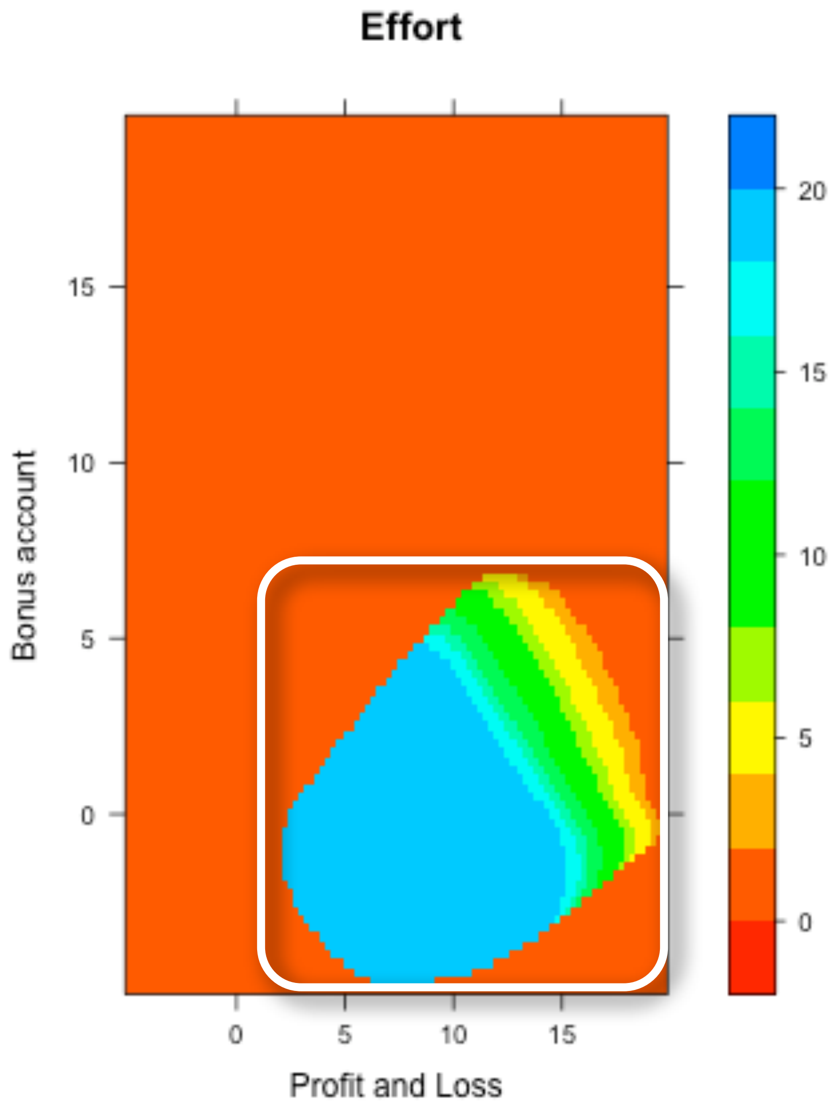


High Utility

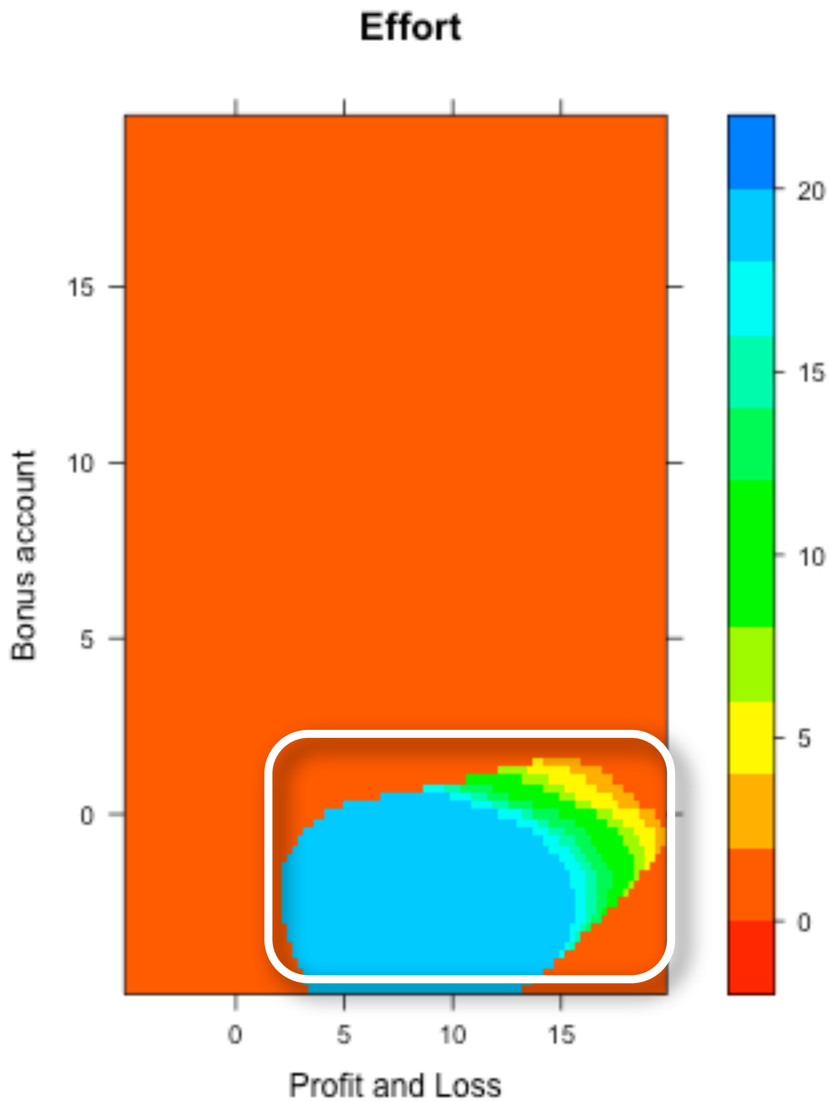


Low Utility

Cheap Effort



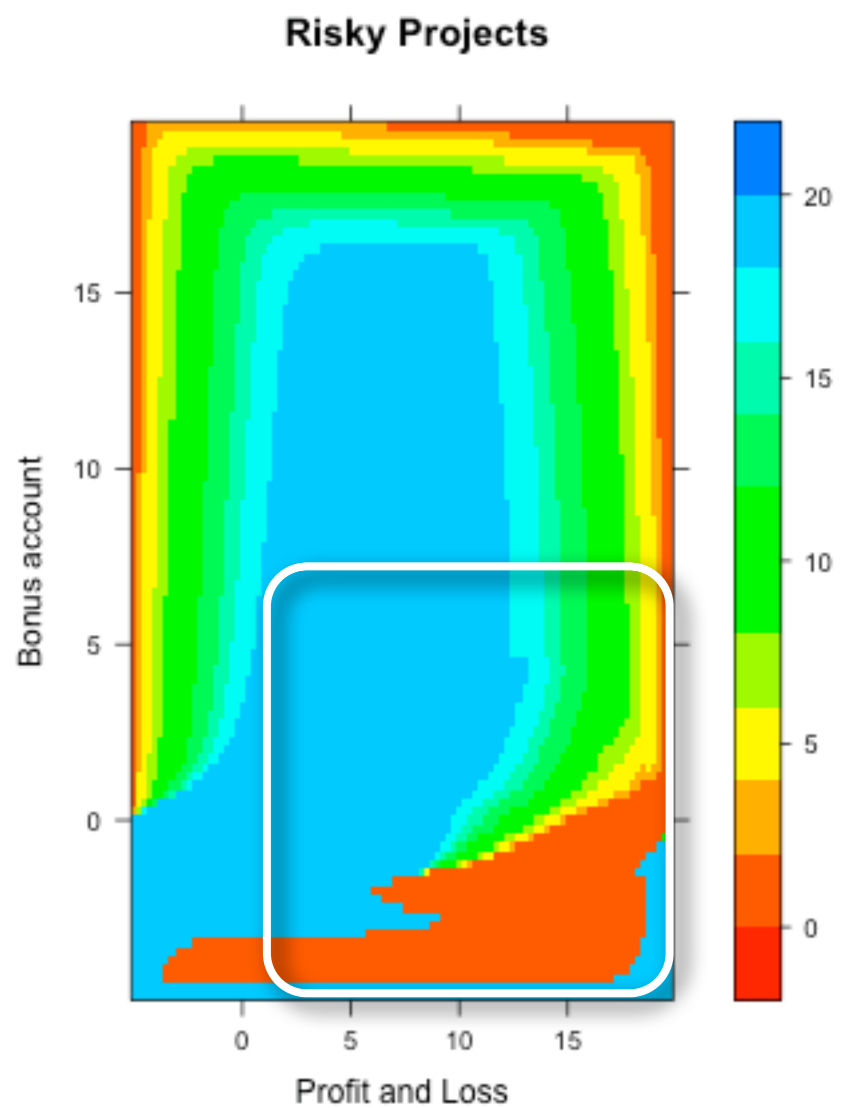
High Utility



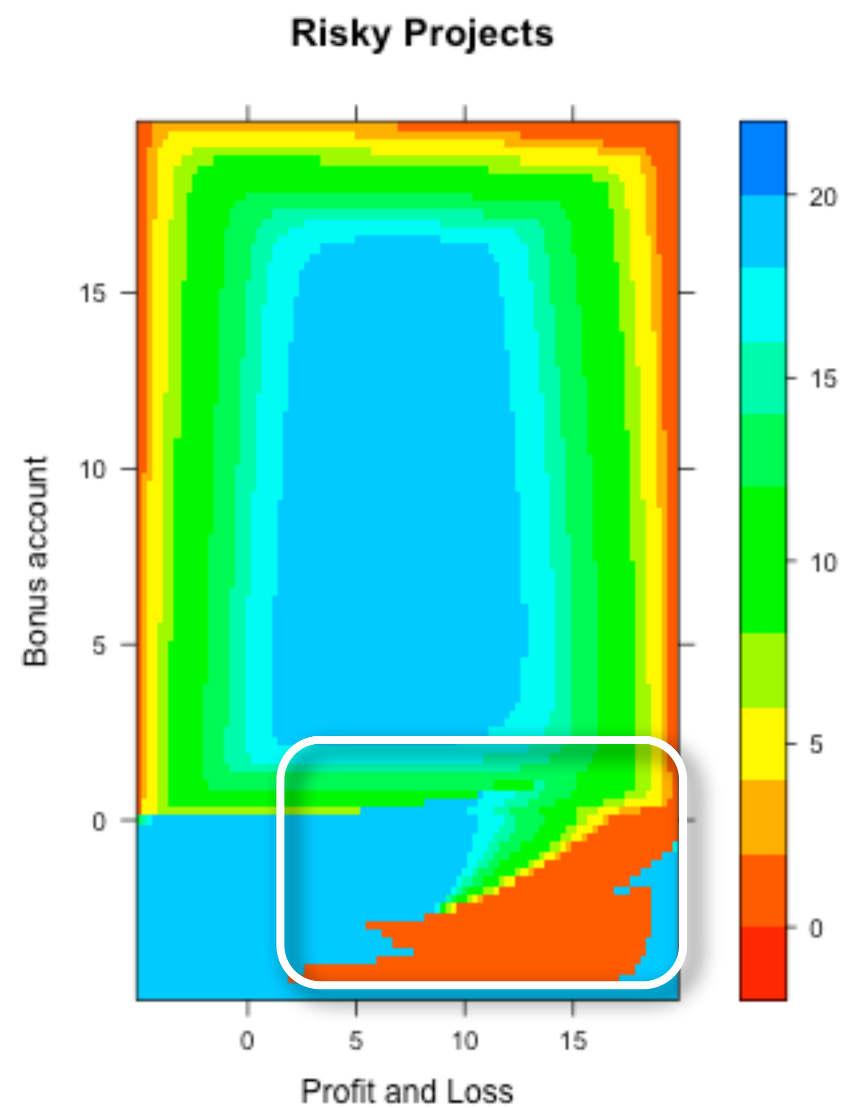
Low Utility

Lower Impact

(cheap effort)



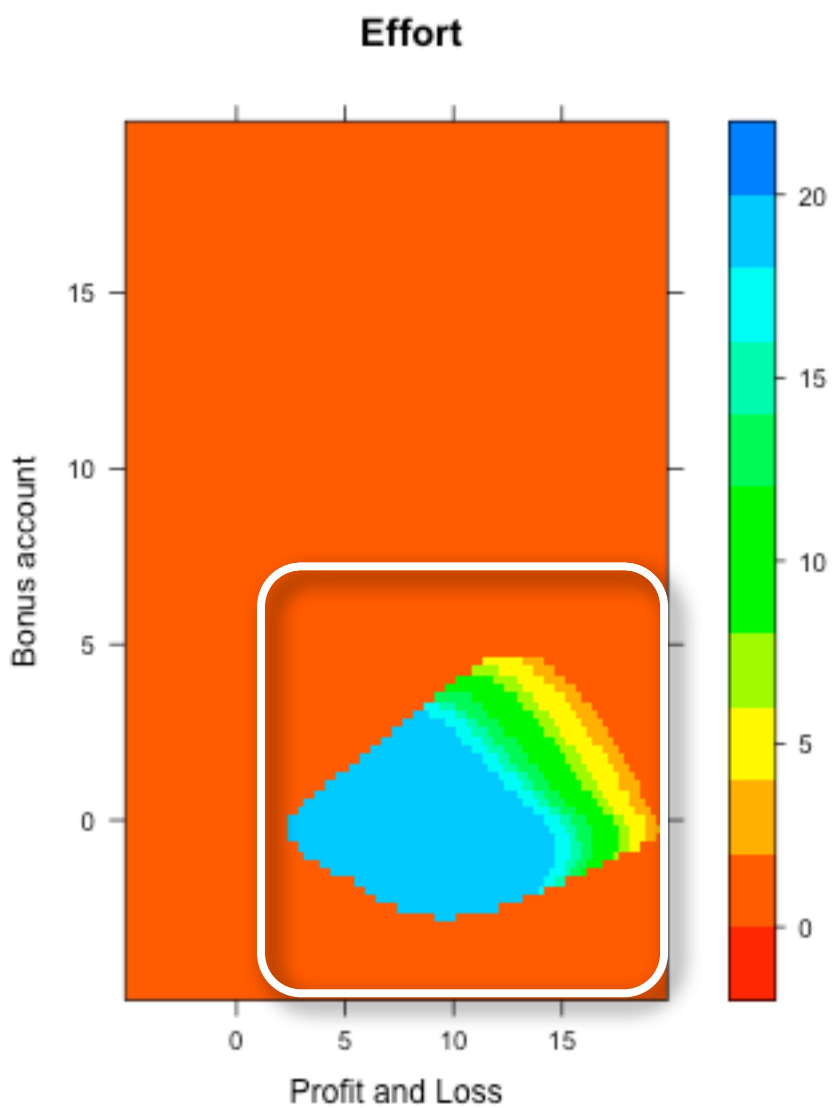
High Utility



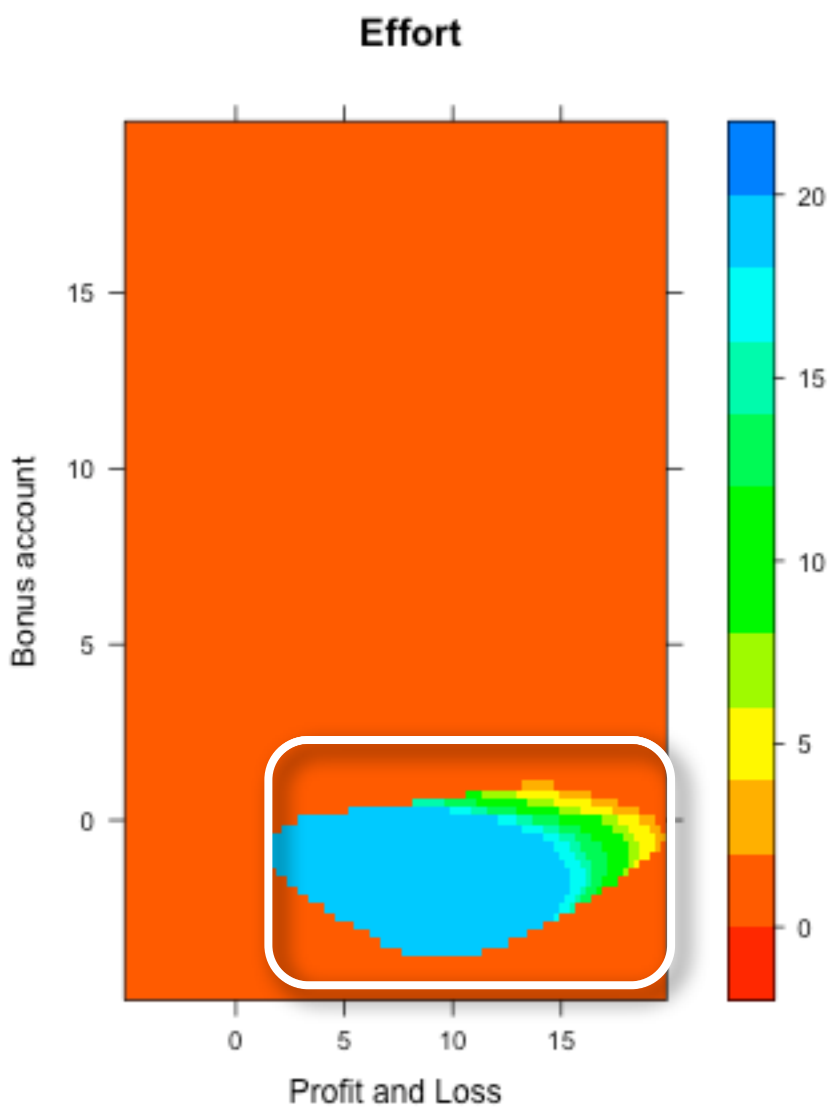
Low Utility

Lower Impact

(cheap effort)



High Utility



Low Utility

Conclusions

(preliminary)

1. Costly effort causes executives to take no effort but only risk.
2. Executives with high utility take higher risk when company close to loss.
3. A negative bonus account cause managers to make effort and also less risk *however* only when company makes profit.

Summary

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(preliminary)

1. To analyse bonus/malus compensation plan use Stochastic Optimal Control.
2. Stochastic Optimal Control permits to state the optimum for a utility maximising executive.
3. Markov Chain Approximation allows to approximate solution.
4. Results show that a bonus/malus cause managers to increase costly effort and reduce free risk.

Questions?

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