# Calibrating Smile and Correlation in a Stochastic Volatility Libor Market Model

**Eurobanking 2009, Peter Caspers**

# Disclaimer

The contents of this presentation are the sole and personal opinion of the author and do not express WGZs opinion on any subject presented in the following.

# Product examples

- Multicallable Swaps 5% vs. Libor
- max(10% - 2 x Libor, 0%) vs. Libor
- CMS5y vs. Libor
- max(CMS10y-CMS2y, 0%) vs. Libor
- Multicallable Range Swaps
- Tarn Swaps
- Multicallable Snowball Swaps

...

# Important features of a „good“ pricing model

- Match today's market prices of liquid plain vanilla hedging instruments, i.e. today's market information about volatilites, correlations and smiles relevant for the payoff

  The price of the exotic is equal to the price of a replicating (hedging) portfolio. If hedge instruments are priced wrong, so is the exotic.

- Imply reasonable future volatilities, correlations and smiles.

  Hedging of an exotic requires dynamic rebalancing of the hedge portfolio. Therefore wrong future hedging in the model implies a wrong price today.

  One common way to achieve this is Time Homogenity of the models parameters.

# Market information to match

- Implied Cap Volatilities including the whole Smile
- Implied Swaption Volatilities including the whole Smile
- (CMS Swap Spread Quotes)
- CMS Spread Options Quotes, possibly for different Strikes

# LMM model specification

Each Forward rate evolves (in its own forward measure) according to (T = maturity)

d Forward(t) = ( *skew* Forward(t) *+ (1-skew) Forward(0)* ) g( t ) sqrt(v) dW

d v(t) = meanreversion ( 1 – v(t) ) dt + volvol sqrt(v) dV

Classic LMM

*Displaced Diffusion LMM*

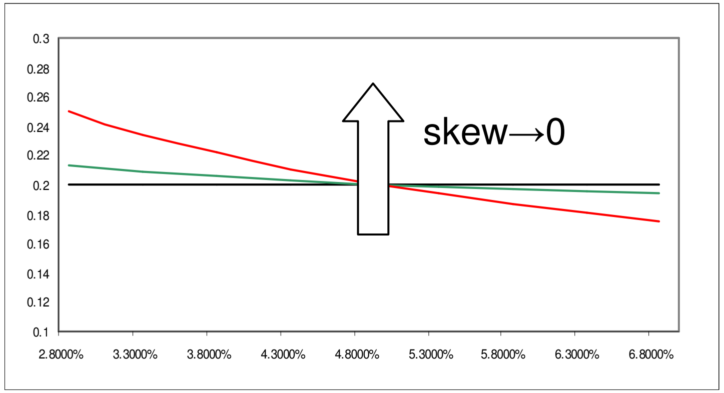
Stochvol Extension LMM

- g is a Forward specific, deterministic instanteous volatility.
- The Forwards Brownian motions are linked via an instanteous correlation matrix.
- v is a scaling factor for the instanteous variance of the forward rates.
- The Stochvol Brownian motion V and the Forwards Brownian motions W are uncorrelated.
- The mean reversion and volvol (volatility of volatility) are constants.

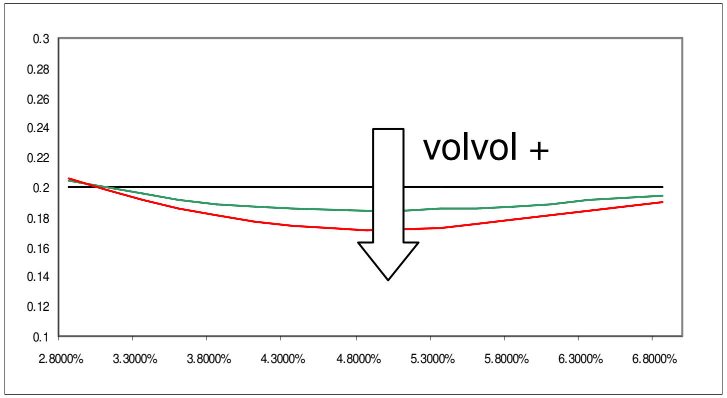
# European option valuation in the stoch vol LMM

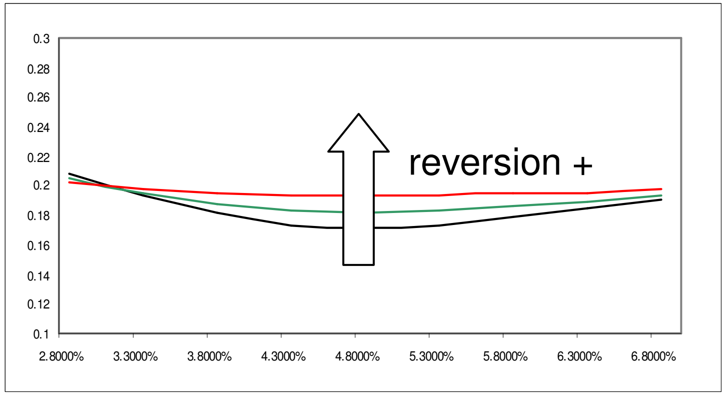
- For calibration purposes it is important to have analytical formulas / approximations for european swaption prices (caplets, swaptions, cmslets, cms spread options).
- Markovian projection methods can be used to derive approximate dynamics for swap rates that look the same as the dynamics for forward rates. (Piterbarg)
- Parameter averaging methods can be used to calculate constant parameters for skew and volatility such that the distribution of the rate in question at a maturity in question is the same as for the model with time dependent parameters. (Piterbarg)
- Transformation methods can be used to compute prices of european options in stoch vol models with constant parameters.
- There is a recent paper with analytical approximations of Cmslets and Cms Spread Options (Antonov, Arneguy: “Analytical formulas for pricing CMS products in the Libor Market Model with stochastic volatility” (Jan 2009)).

# Parameter impact on smile

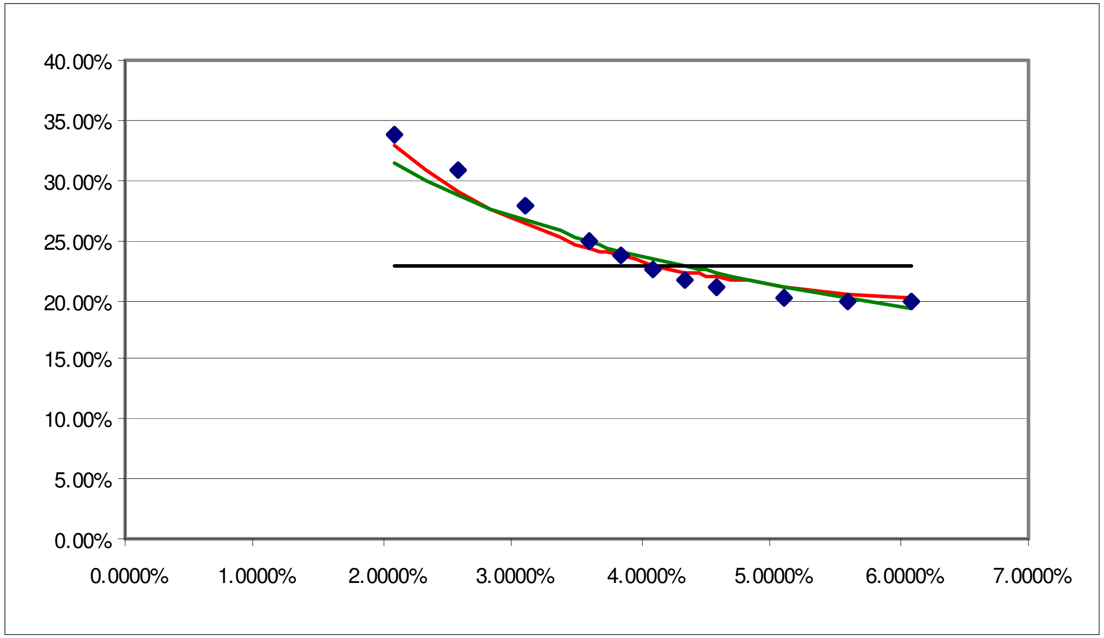



- displacement affects skew of smile
- volvol and reversion affects curvature



# LMM Smiles




Classic LMM

5y/2y Swaption as of 09-12-08

*Displaced Diffusion LMM*

Stochvol Extension LMM

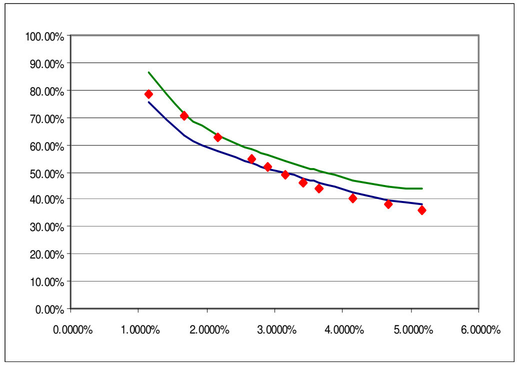
# Calibration: Full Time Homogenity

- We use a Time homogeneous instanteous correlation matrix C = C ( T(i) – t, T(j) – t ) for Forwards i and j. At this stage we use simply C ( x, y ) = exp( - decay | x – y | ).
- We also use a Time homogeneous instanteous volatility g = g ( T(i) – t ) for each Forward i. We use the popular abcd – functional g(x) = (ax+b) exp(-cx) + d.
- We fix the mean reversion (50%) and volvol (170%), i.e. we do not calibrate these parameters.
- We also fix the correlation decay parameter at this stage.
- Thus we are left with a model with 4 free parameters (a,b,c,d).
- Our calibration instruments are swaptions with option tenors 6m, 1y, 2y, ... , 10y and underlyings swap rates 2y, ... , 10y including their smiles.
- The calibration is done using a simulated annealing algorithm build upon the Nelder Mead procedure.
- The objective function is the RMSE of relative errors in terms of implied Black volatilities with all calibrating swaptions equally weighted.

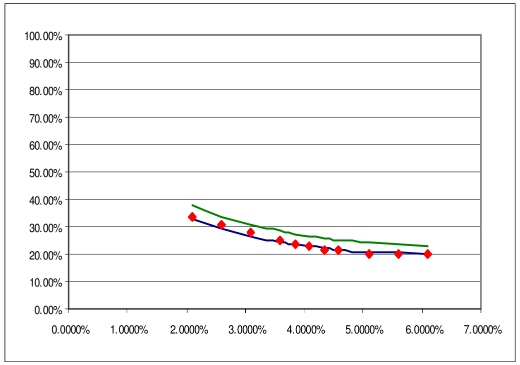
# Calibration results (Full Time Homogenity)

market data as of 09-12-08

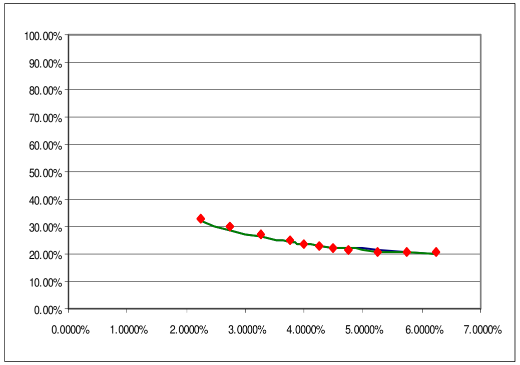
- dots = market
- blue = simple SV
- green = full LMM



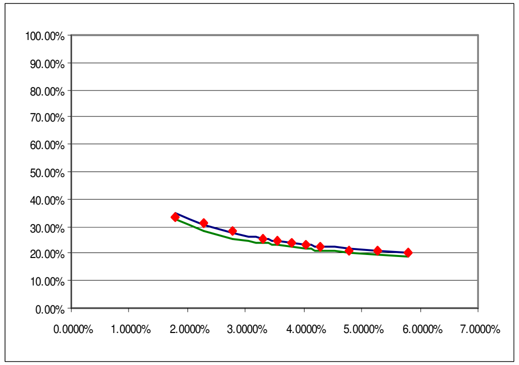

6m/2y




5y/2y




5y/5y



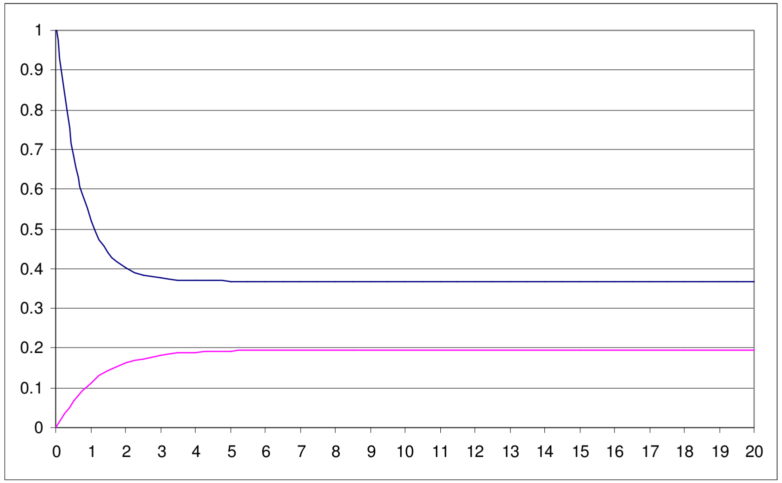

10y/10y

# Calibration results (Full Time Homogenity)

- Calibration Error of around 10% RMSE.
- Problems with short dated options and short maturity underlyings
- All in all not satisfying.

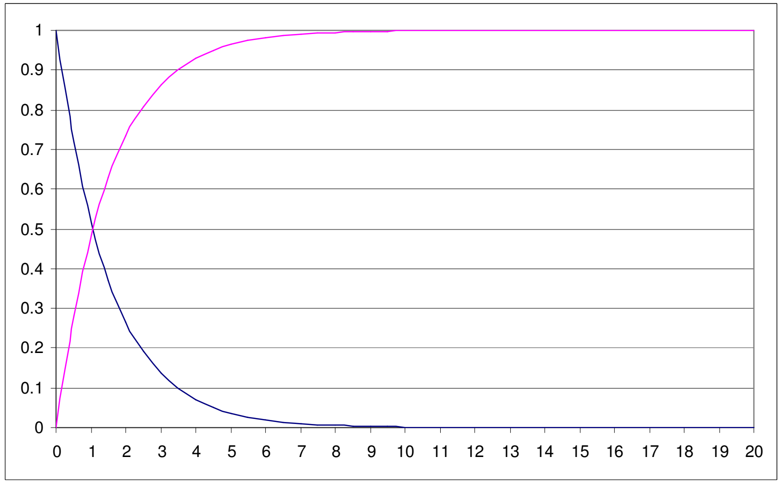
# Calibration: Two state approach

- Using an idea of Rebonato, we use two states of volatility expressed by two abcd functionals g1 and g2. g1 represents the “excited” state, g2 the “unexcited” state.
- The models volatilty function is then defined as $g := \exp(-\lambda t)\, g1 + (1-\exp(-\lambda t))\, g2$ with a decay parameter $\lambda$. This is no longer a time homogeneous approach. Instead, an intentional inhomogenity is introduced. The idea is, that the model starts in an “excited” state and cools down to a “unexcited” state after some time. The parameter $\lambda$ describes how fast this transition takes place.



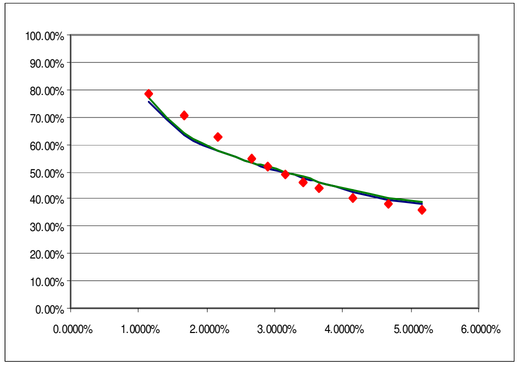

excited state (blue), normal state (red)

calibration on data as of 09-12-08

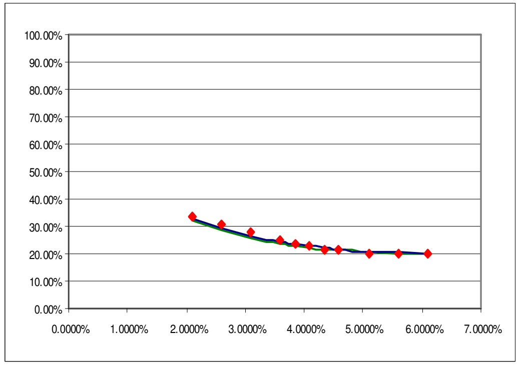



weight functions

# Calibration results (Two State approach)



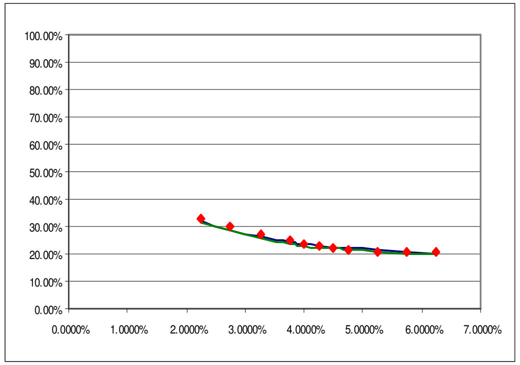

6m/2y



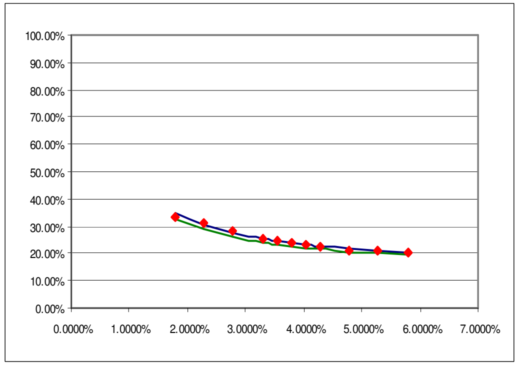

5y/2y

market data as of 09-12-08

- dots = market
- blue = simple SV
- green = full LMM




5y/5y




10y/10y

# Calibration results (Two state approach)

- Calibration Error of around 3% RMSE (full time homogeneous case: ~10%)
- Good calibration of short dated options and short maturity underlyings
- All in all good fitting.

# CMS Spread Options

- Important calibration instruments when a CMS Spread is involved in the payoff
- Very sensitive to rate correlations (compared to swaptions)
- Analytical Approximations in stoch vol models were developed recently. Since our implementation is not yet complete, the following results are taken from a displaced diffusion model without stochastic volatility.
- With a time homogeneous correlation structure, it is hard to match Spread Option Premia for all maturities. Therefore we used a modified version of

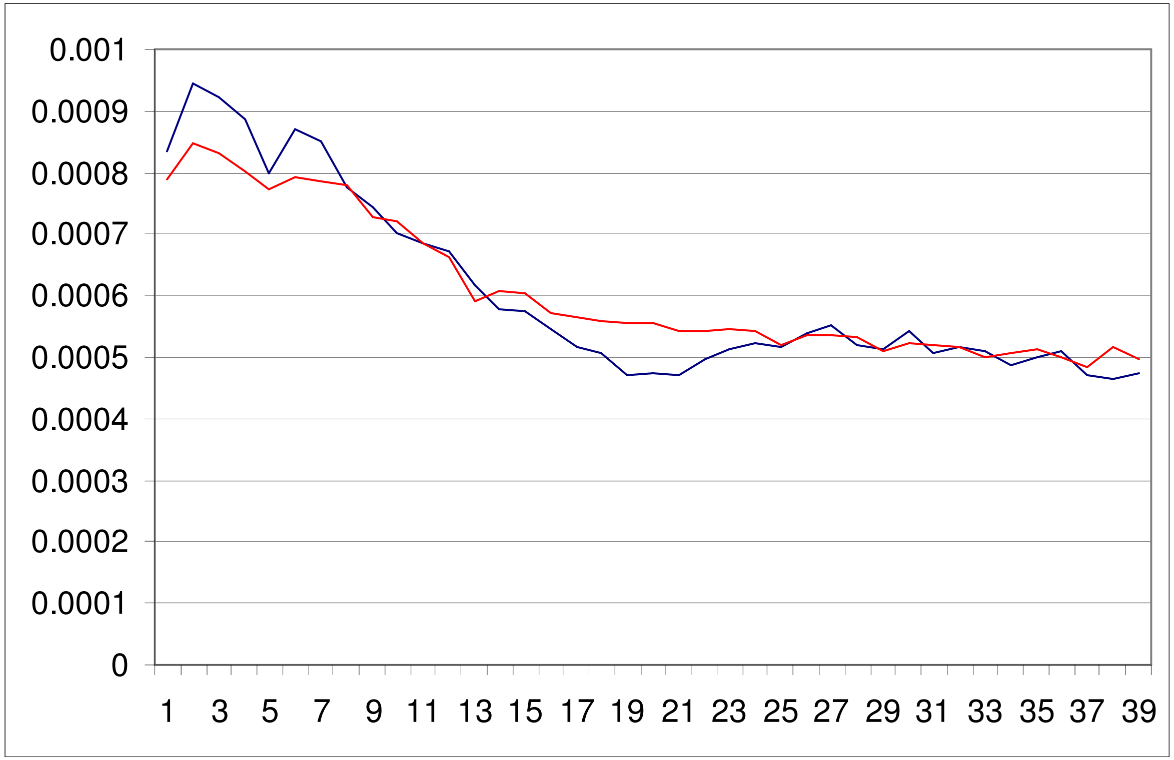
  $C(x , y) = rho_{inf} + (1-rho_{inf}) \exp ( - decay \mid x - y \mid )$

  where decay is no longer a constant but

  $decay ( t ) = d_{inf} ( 1 - \exp ( - \lambda \ t ) ) + d_0 \exp ( - \lambda * t)$

- Note that this can be interpreted as a "two state approach" for the correlation structure, similar to the volatility case earlier.

# CMS Spread Options – Calibration results




market data as of 26-06-08, blue = strip from market premia, red = model premia

# CMS Spread Options

- We need a two state approach to get a satisfying match of all spread option premia.
- One can express the spread optionlet premia by the implied correlation parameter in a simple lognormal reference model. Thus one gets an implied correlation surface in the dimensions maturity and strike level of the option.
- One observes, that the implied correlation shows an inversed smile shape. By that the market corrects the bivariate normal assumption of the reference model.
- In the displaced diffusion LMM, we observe flat correlation smiles instead, i.e. we can only calibrate to one strike level at a time and not fit the whole correlation smile.
- This is relevant for the pricing of certain payoffs, e.g. Range Accrual Swaps on Cms Spread (with two conditions) or Tarn Swaps on Cms Spread.