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Risk scenarios by means of PCA

A crude framework for the generation of risk scenarios by means of principle component analysis

Eurobanking 2007, Glion sur Montreux

Agenda

- Motivation
- The framework
  - Principal Component Analysis
  - Identification of extreme interest rate movements
  - Generation of scenarios
  - Historical representatives

Motivation

Management of funding portfolios for variable rate products ...

Typically

– base scenarios (likely):	research, consensus, forward, constant
– risk scenarios:	base scenario ± parallel shifts, steepening, flattening BIZ standardised shock scenarios (±200 bp instantaneously)

Criticism of setup for risk scenarios generation

- no model for risk scenarios
- yield curves don't necessarily move that way
- steepening/flattening risk scenarios are user dependent
- operational risk

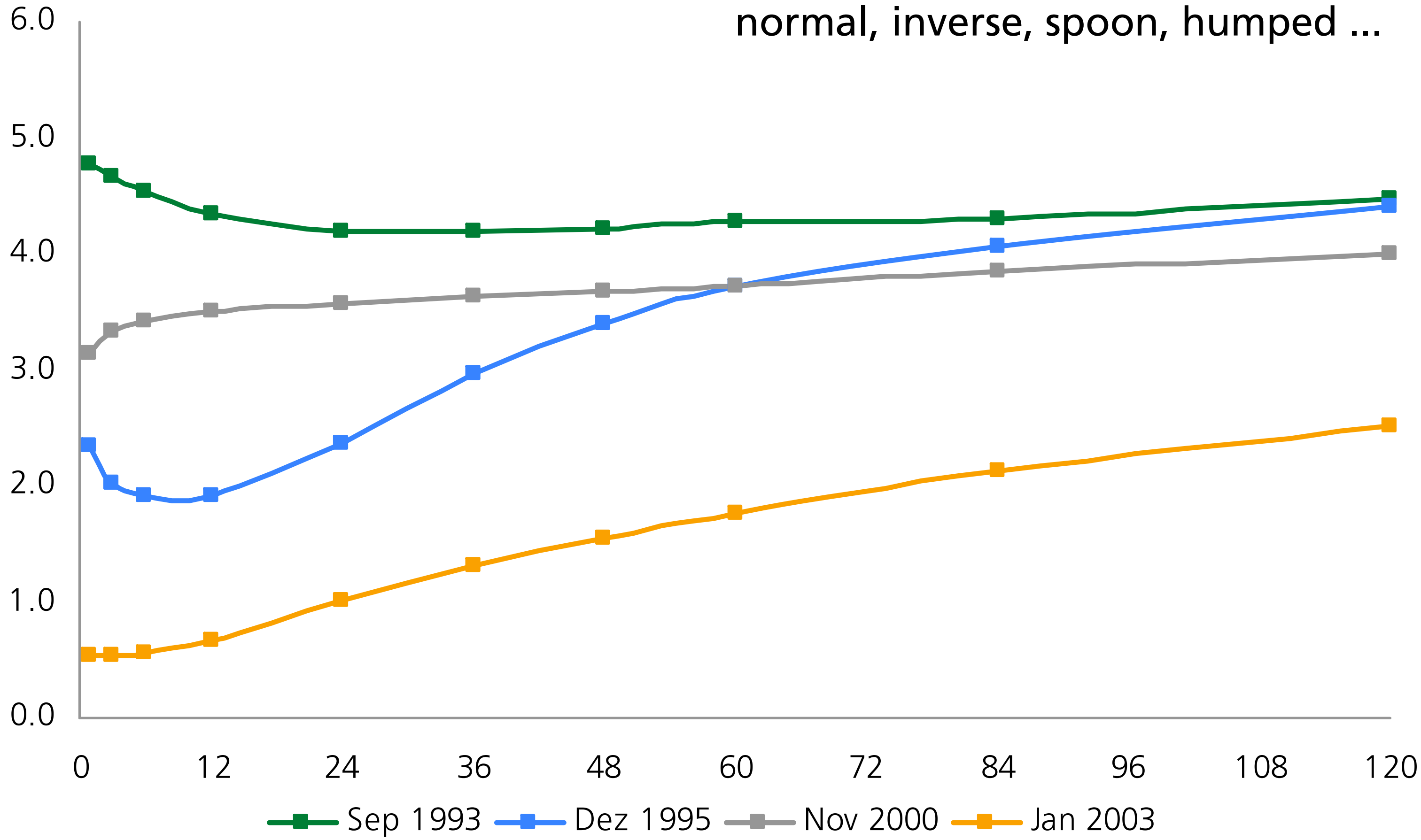
... is based on scenario analysis

Resolve the disadvantages of current risk scenario generation

- How do yield curves move in reality?
- Can we describe interest rate movements systematically?
- Is it possible to generate risk scenarios systematically?
- How can we reduce user dependence and operational risk?

Intuition

– Interest rate movements show high correlations
– exhibit few yield curves shapes: normal, inverse, spoon, humped ...



→ Determine the most important 'modes of variability'.

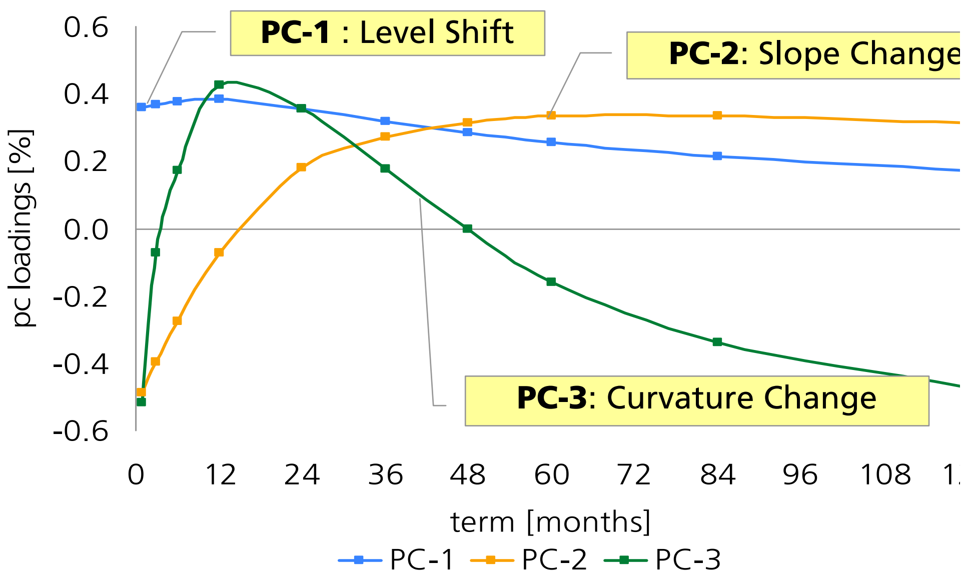
Principal Component Analysis – how to do it

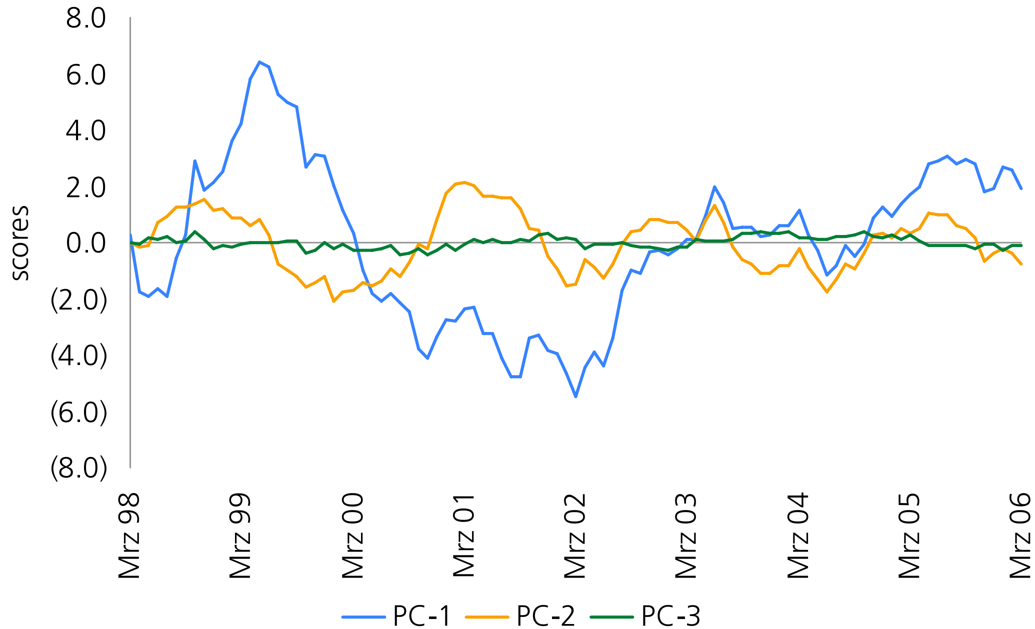
- Input: historical interest rate time series for a list of tenors
  - $\mathbf{R}_{t,m}$ (t=date$_1$, date$_2$, …, date$_n$; m = 1, ..., p (e.g. M3, M6, ..., Y10))
- Calculate differences (and center) them: $\mathbf{X} = (\mathbf{R}_{(t+L), m} - \mathbf{R}_{t, m})$
  - L is timeshift (e.g. 1 month movements)
- Evaluate variance covariance matrix $\mathbf{S} = 1/(n-1)\ \mathbf{X}^T \mathbf{X}$
- Solve eigenvalue problem: $\mathbf{S}\ \mathbf{E} = \mathbf{E}\ \Lambda$
  - $\mathbf{E}$ is matrix of eigenvectors $\mathbf{E}=(\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_p) \rightarrow$ PC vectors
  - Λ is matrix of eigenvalues $\Lambda=\mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_p) \rightarrow$ variance of PCs
- Shuffle the eigenvectors according to decreasing eigenvalues
- Extract the representation of $\mathbf{X}$ in terms of PC vectors $\mathbf{A} = \mathbf{X}\ \mathbf{E}$
  - $\mathbf{A}$ are principal component scores $a_i(t) = A_{t,i}$ (i = PC$_1$, PC$_2$, …, PC$_p$)

cf. Robert R. Bliss: Movements in the term structure of interest rate (1997)

PCA on CHF Transfer Rates

- Cash Transfer Mid Rates (03/1998 – 03/2007), annual differences





→ Most important modes are change in level (86.7%), slope (12.5%) and curvature (0.4%)

→ For scenario generation we'll restrict to level shift and slope change

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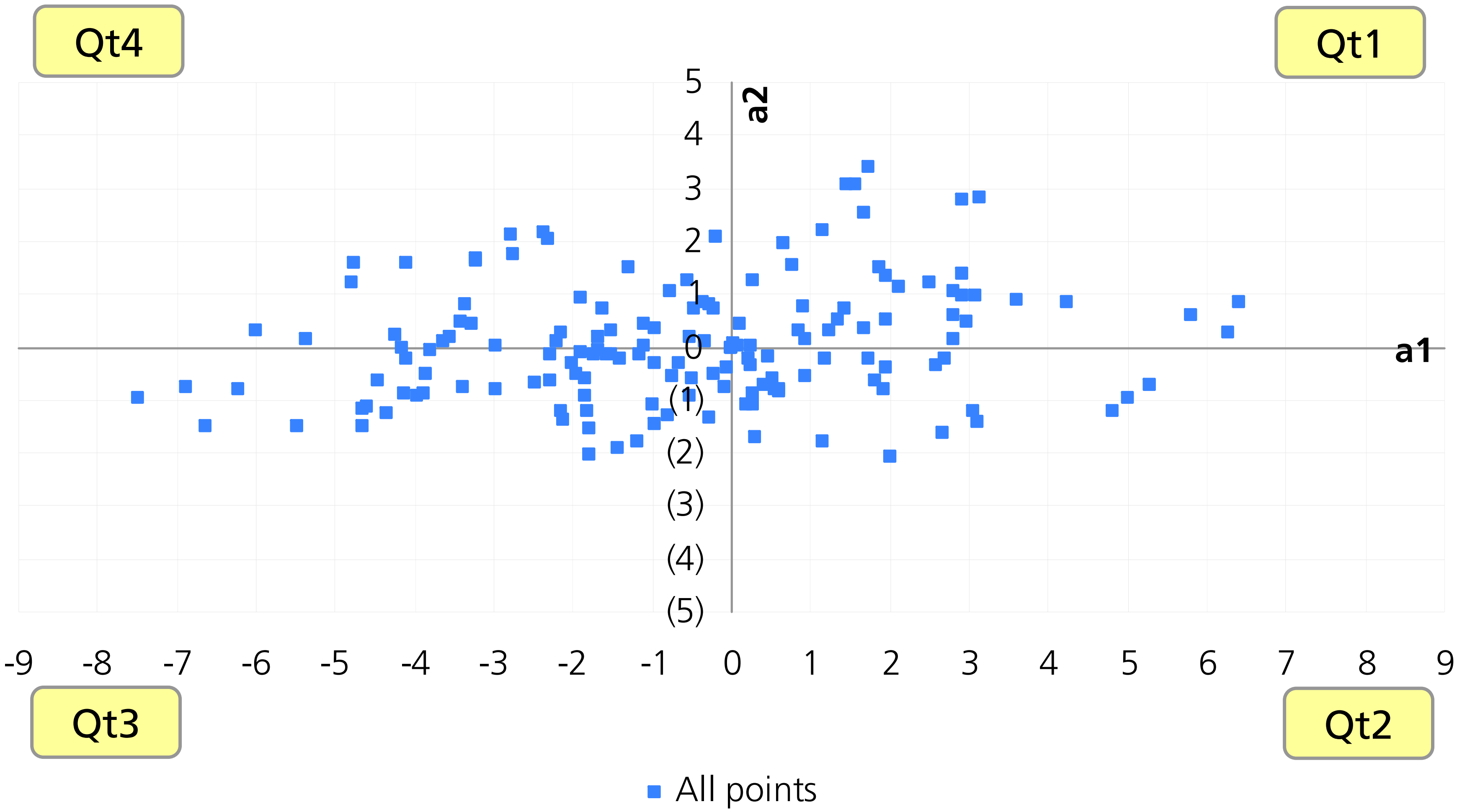
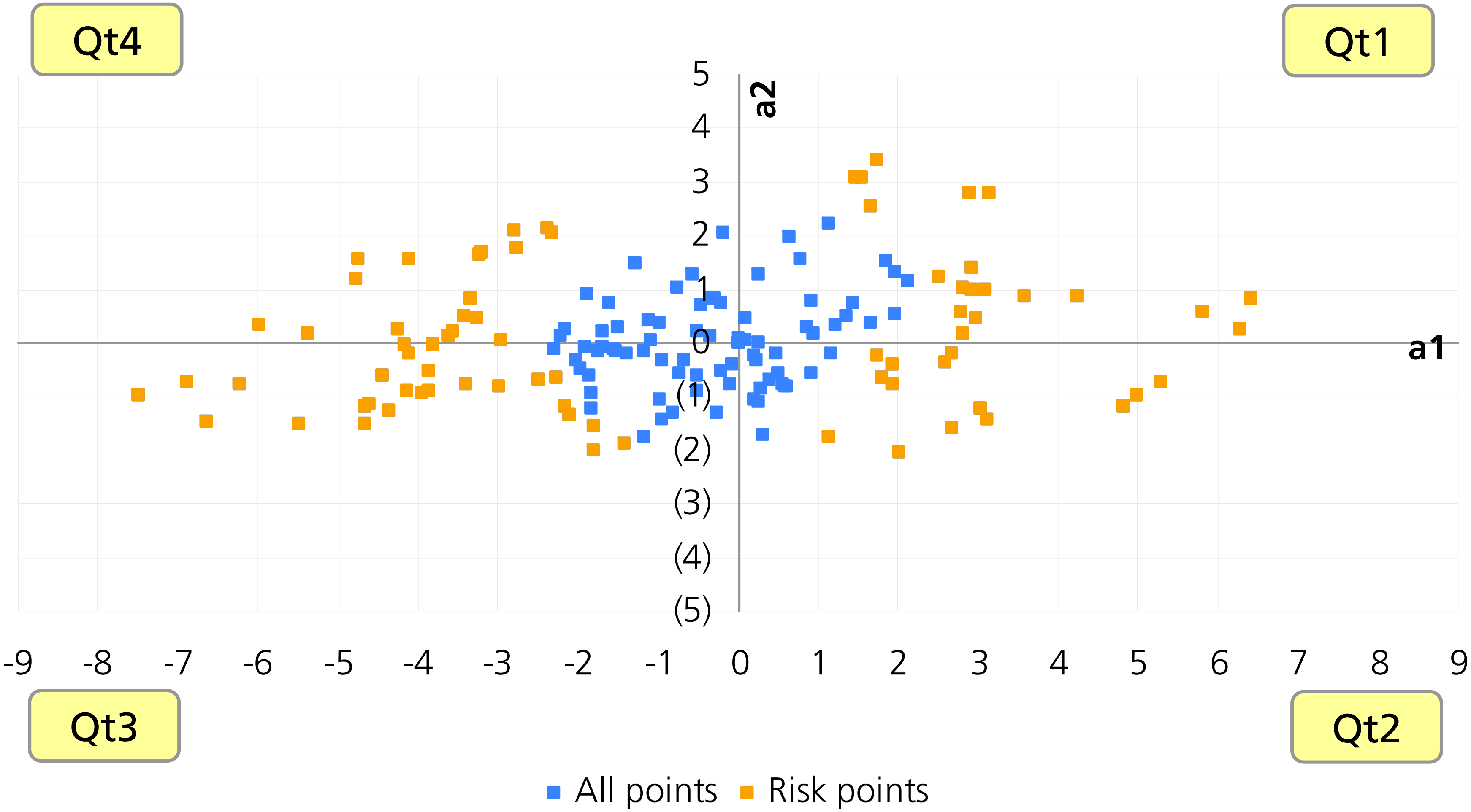
Extreme interest rate movements

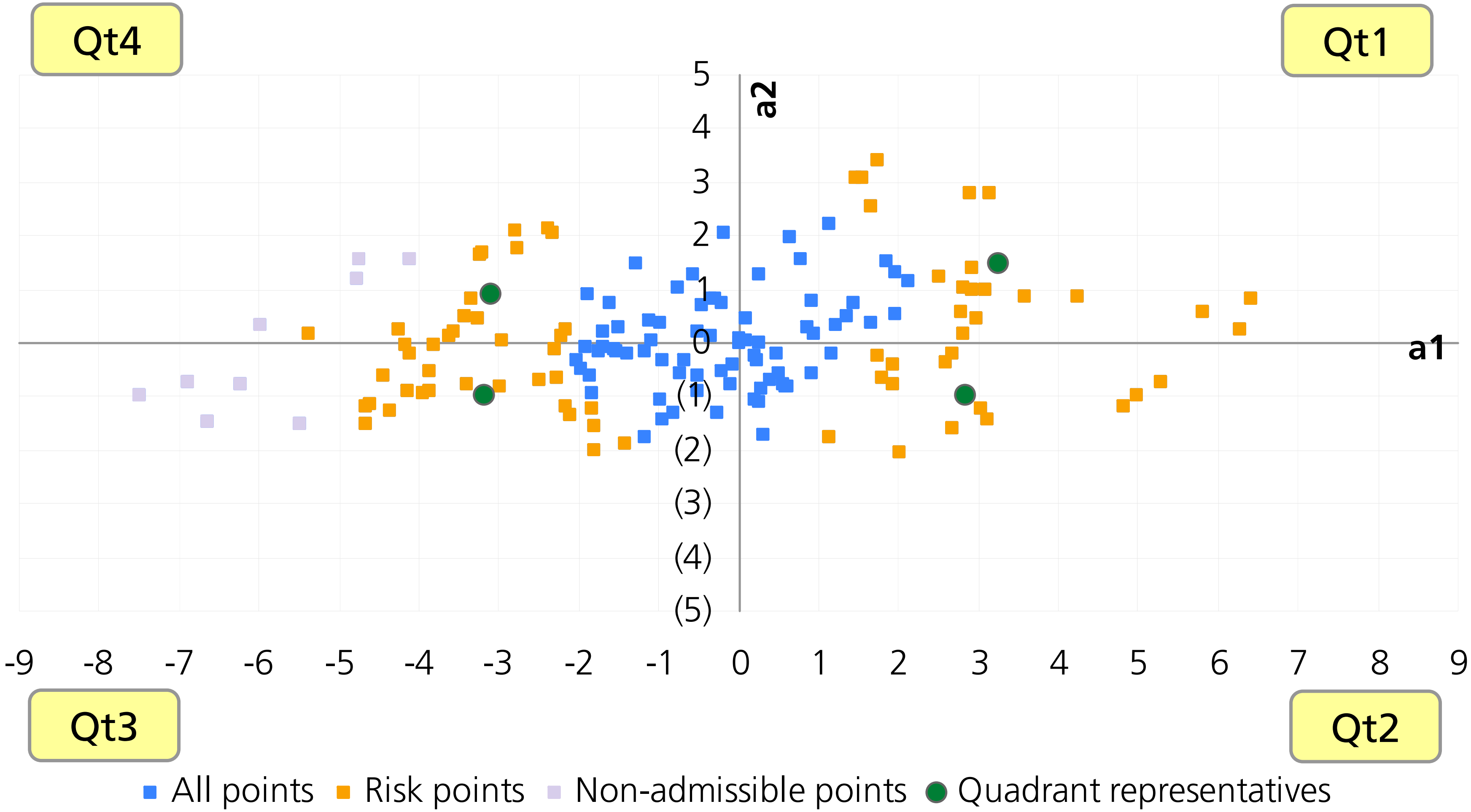
Look at historical scores a1 and a2 for PC1 and PC2



Identify risk points (e.g. 50% quantile in every quadrant)

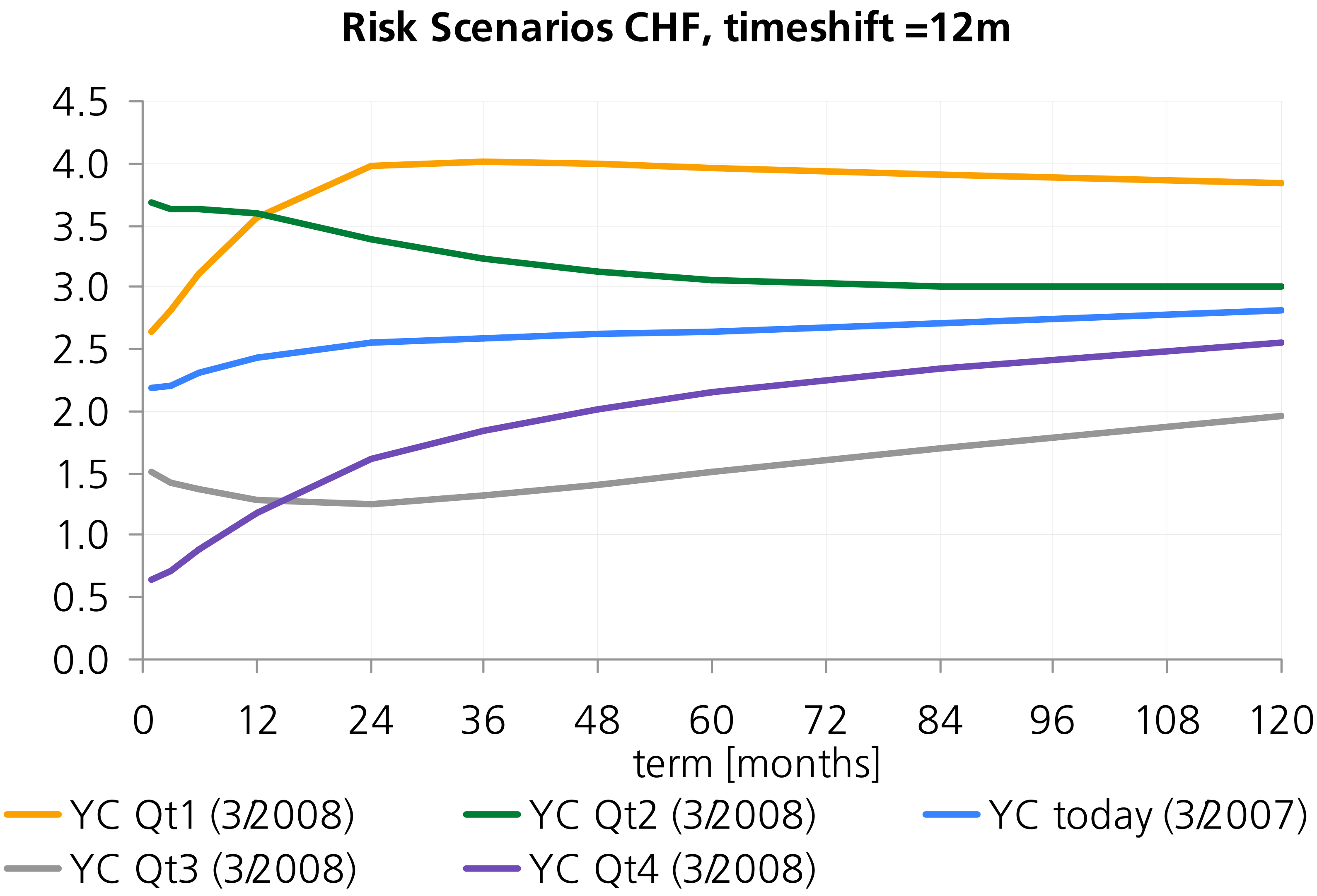


Discard non-admissible risk points (→ non-negative rates) and find centres of mass



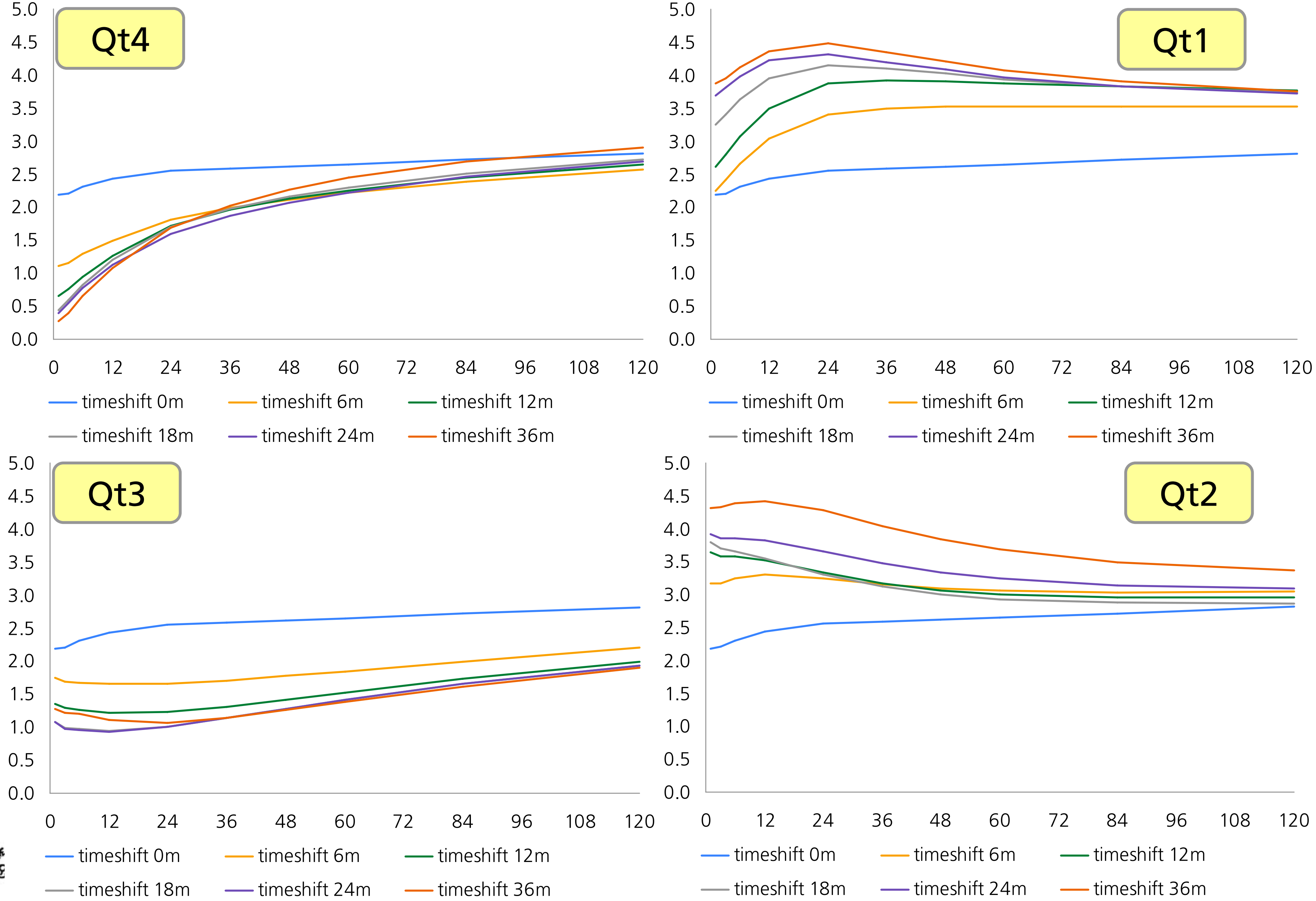
Extreme interest rate movements

Calculate the interest rate shifts for the four centres of mass

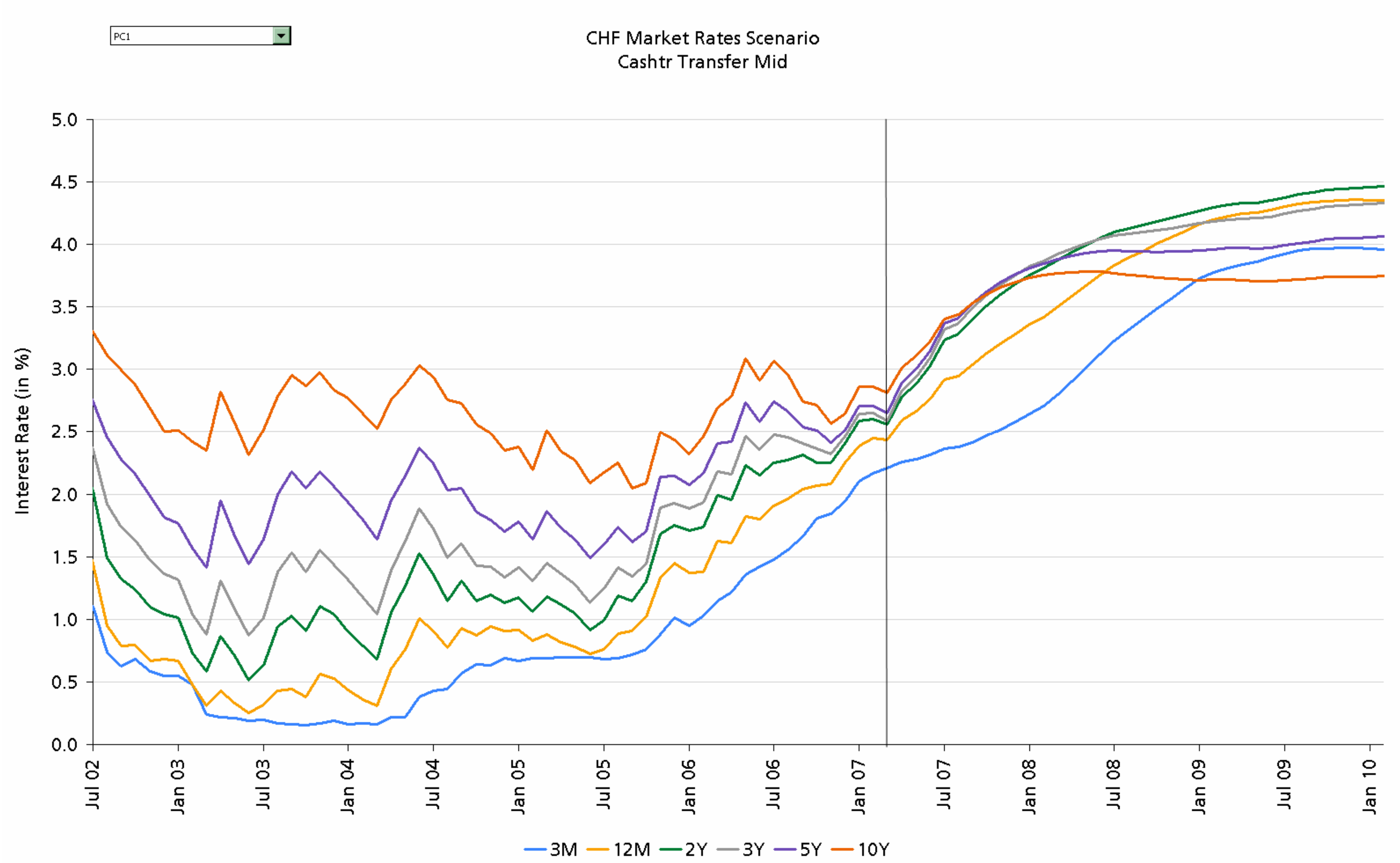


Generation of risk scenarios

Repeat above PCA for varying timelags t = 1m to 36m & smooth



PCA risk scenario Qt1: up steepening



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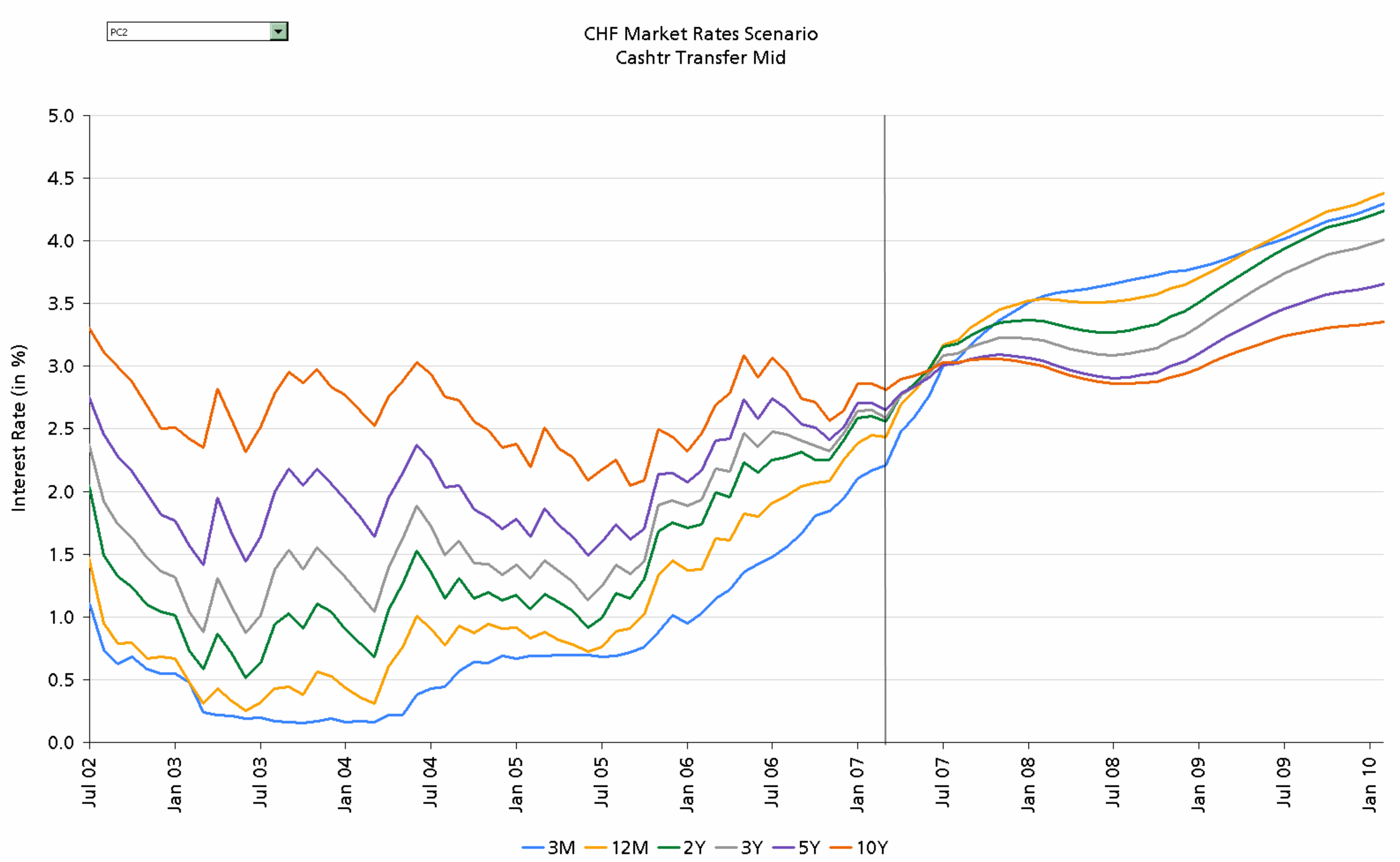
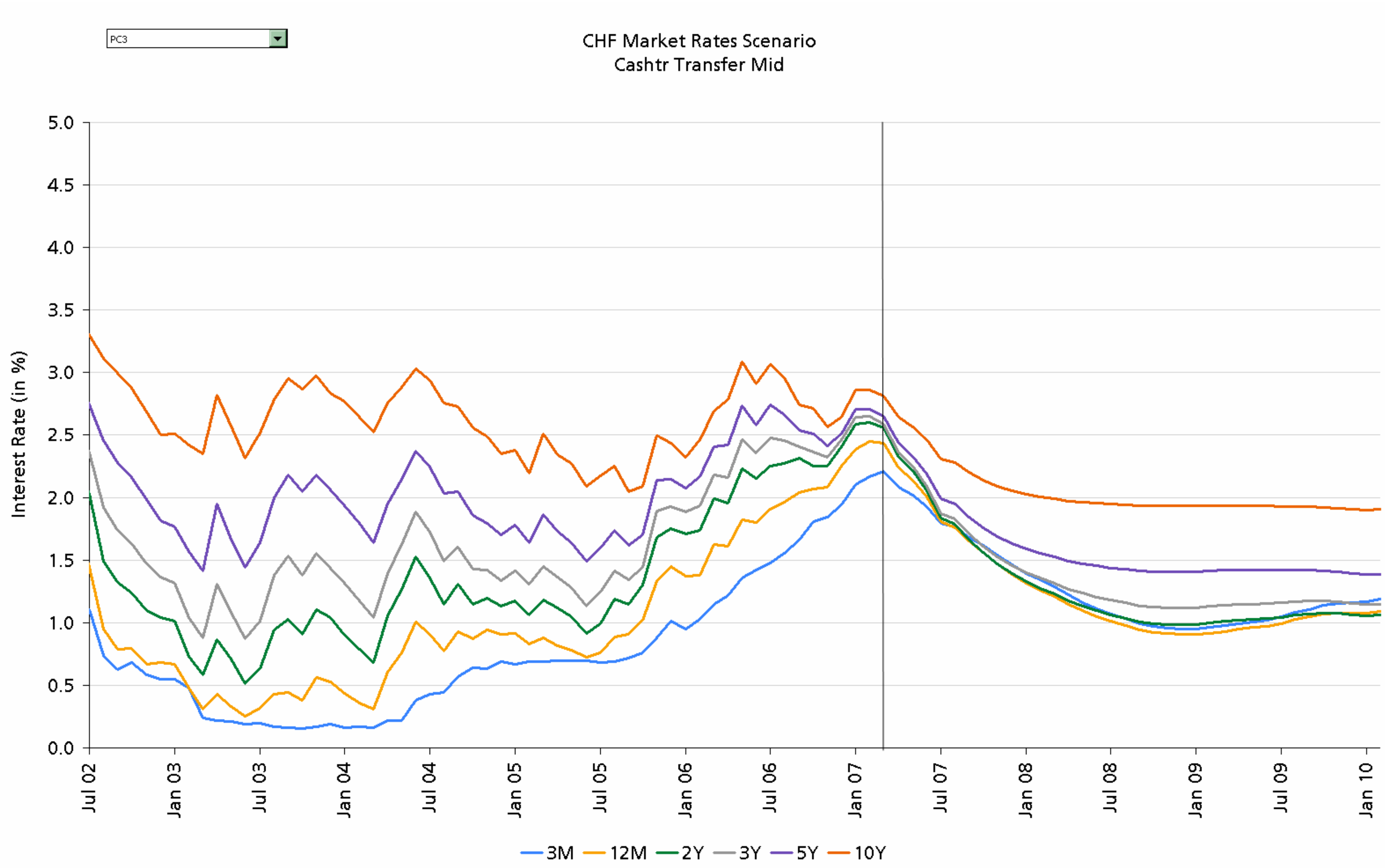
PCA risk scenario Qt2: up flattening/inverse



PCA risk scenario Qt3: down flattening



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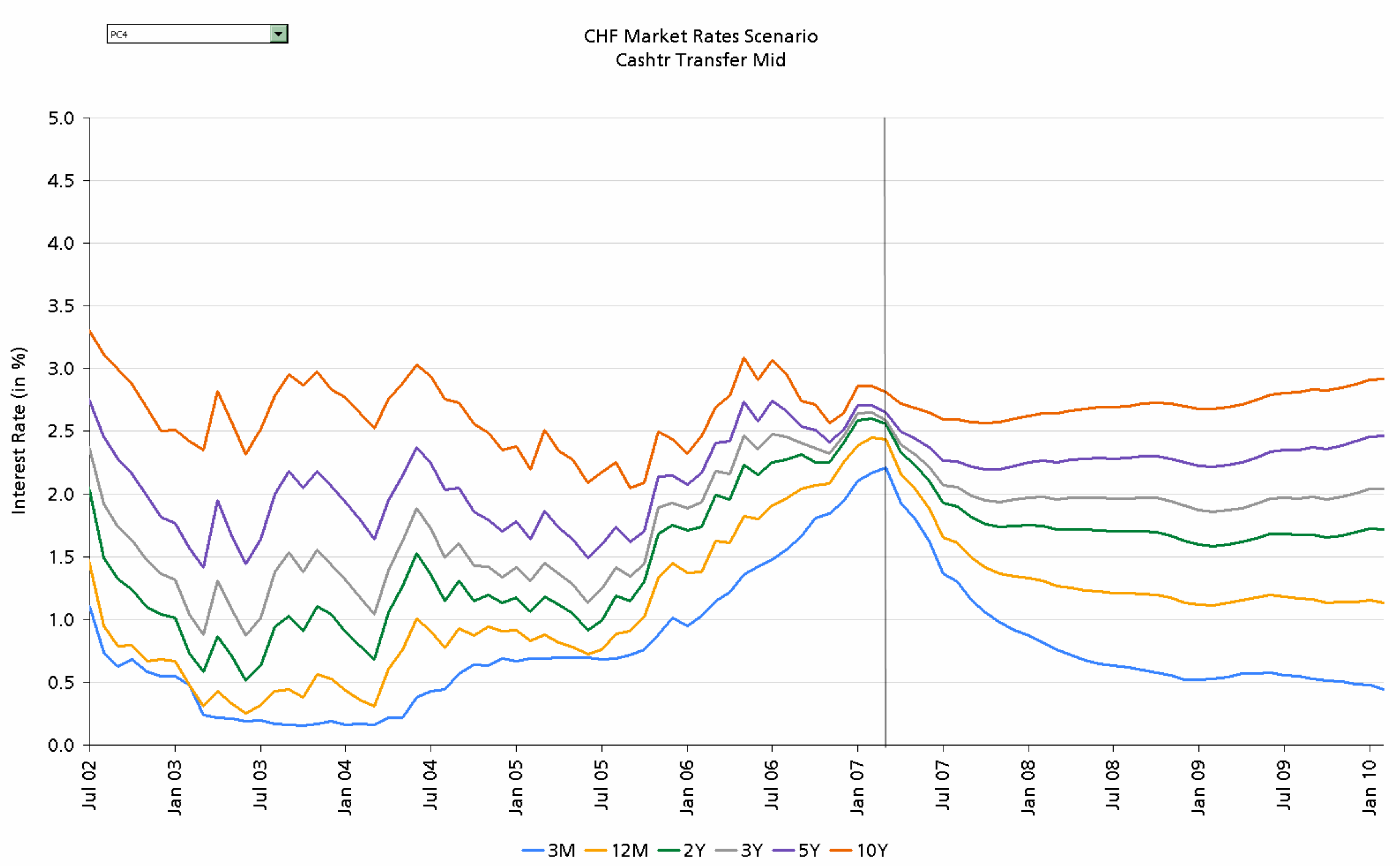
PCA risk scenario Qt4: down steepening



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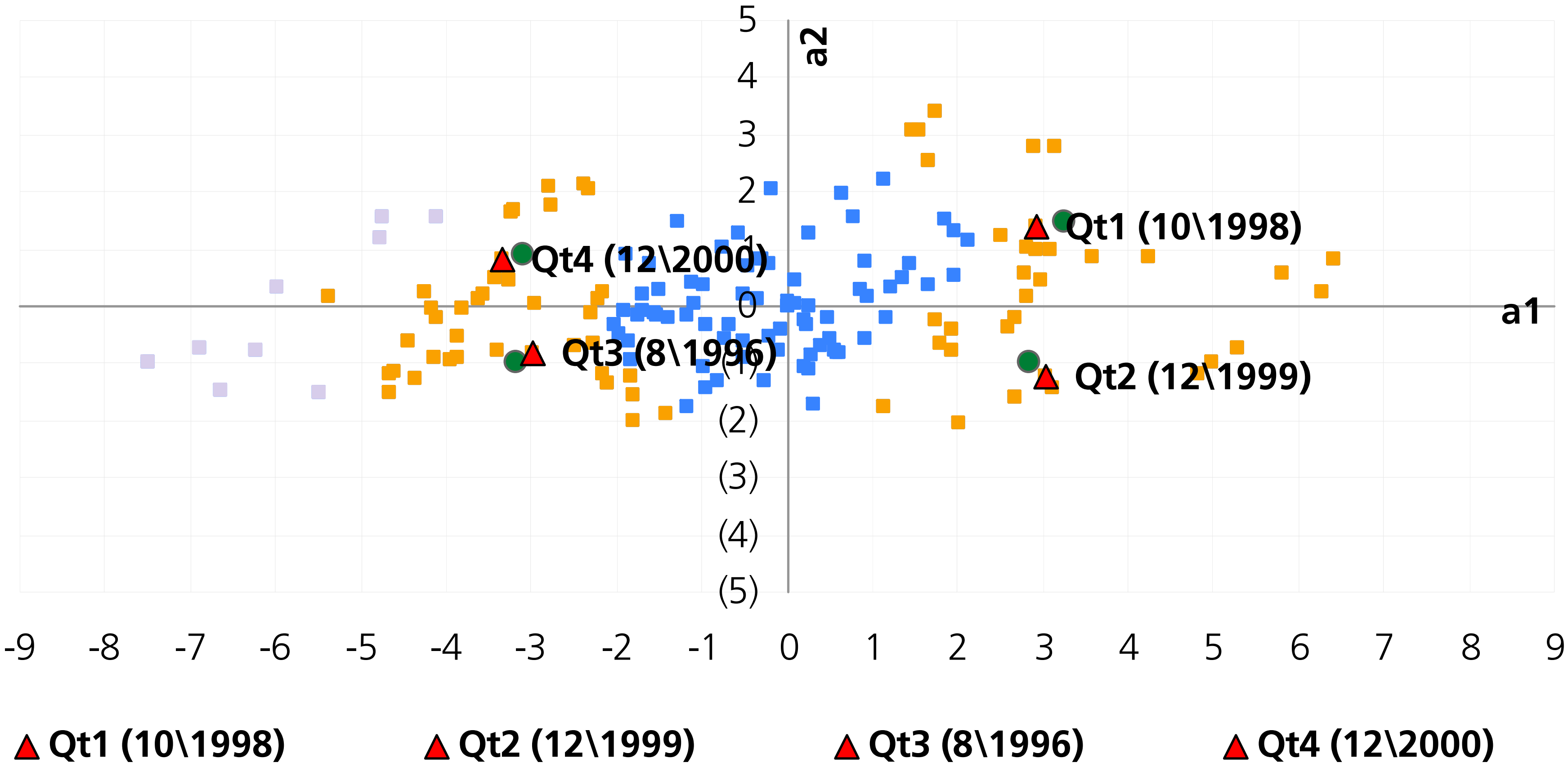
Historical representatives

- Management: "Are your risk scenarios realistic?"
- Answer: "Well, at least we can find historic representatives. For every timeshift there is a point in history when we observed a yield curve shift that is close by."



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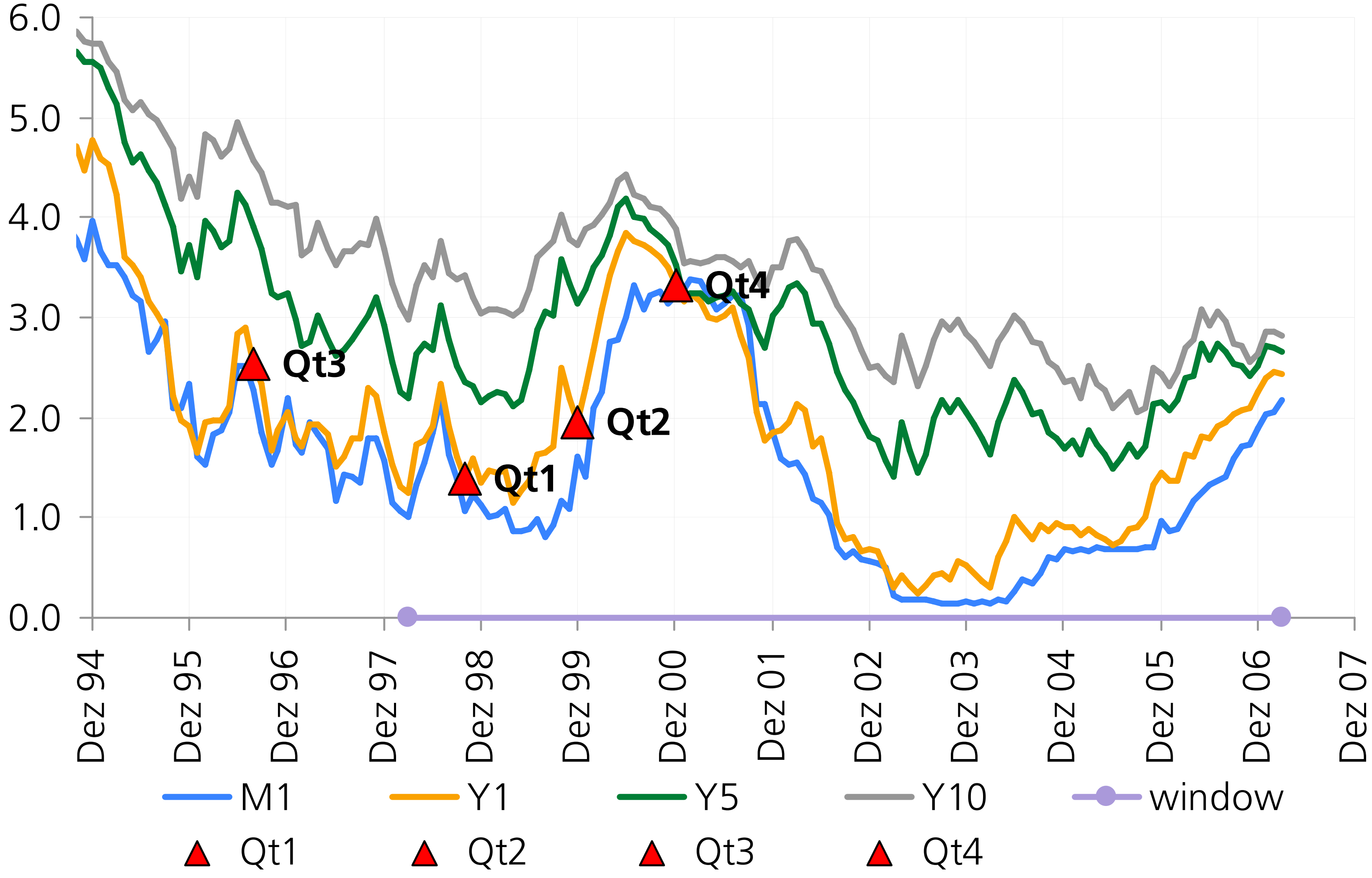
For timeshift = 12m and a 50% quantile ...



Advantages of PCA Frameworks

PCA risk scenarios are superior to former risk scenarios

- relatively easy communication to management
- quantitative model
  - based on historic interest rates
  - reproducible and user independent
- widely automated
- manual input reduced to few parameters
  - time span
  - quantile for scores
  - smoothing parameters
- 'natural' form and dynamic of yield curve independent of parameter choice

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