

Two litte notes on interest rate derivatives gamma computation

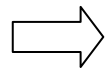
Eurobanking 2010, Peter Caspers



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Agenda



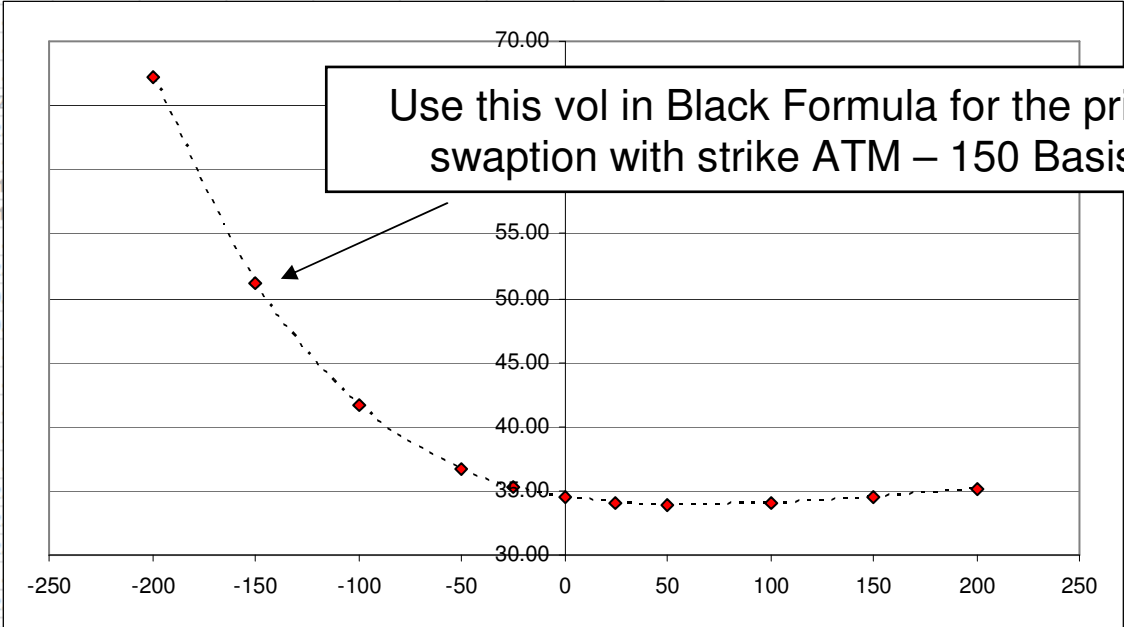
Gamma Computation with Smile

Cross Gammas

Swaption Smile

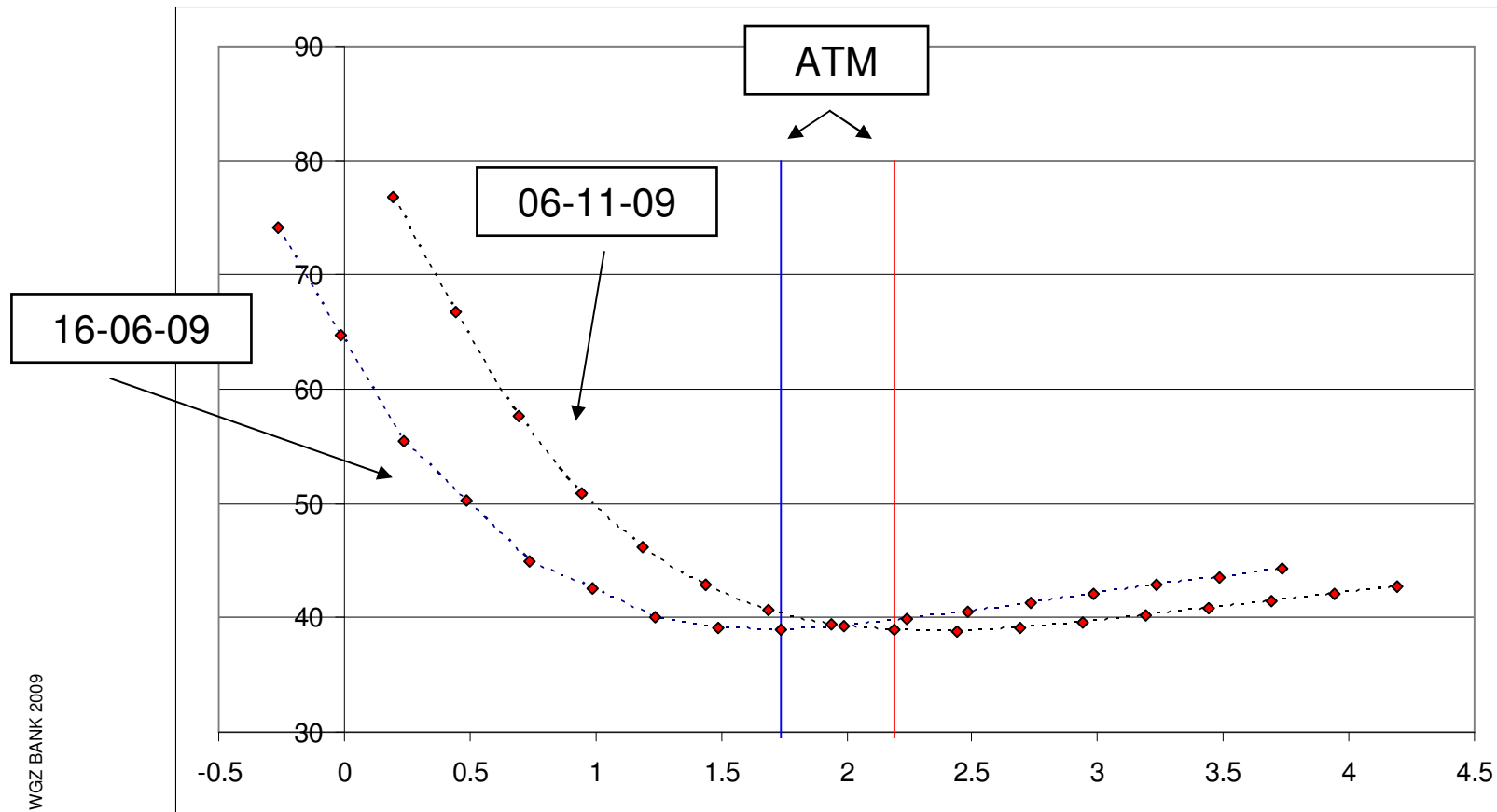
EUR Vol Skews (Vega) float window

	Receivers					Payers					
	-200	-150	-100	-50	-25	ATM	+25	+50	+100	+150	+200
1y2y	32.67	16.62	7.16	2.22	0.84	34.48	-0.45	-0.62	-0.46	0.03	0.64
1y5y	19.96	11.71	6.1	2.3	0.97	24.93	-0.65	-1.04	-1.2	-0.9	-0.38
1y10y	13.82	8.77	4.87	1.95	0.85	19.41	-0.61	-1	-1.23	-0.99	-0.53
1y20y	15.94	10.51	6.09	2.55	1.13	18.15	-0.81	-1.3	-1.44	-0.93	-0.16
1y30y	18.43	12.23	7.2	3.11	1.42	19.89	-1.11	-1.89	-2.52	-2.3	-1.69
2y2y	20.98	11.46	5.58	1							
2y5y	15.4	9.52	5.12	1							
2y10y	12.52	8.05	4.54	1							
2y20y	13.65	9	5.23	2							
2y30y	16.08	10.66	6.27	2							
5y2y	10.31	6.51	3.56	1							
5y5y	9.56	6.09	3.35	1							
5y10y	9.89	6.47	3.68	1							
5y20y	11.1	7.35	4.28	1							
5y30y	13.41	8.88	5.21	2							
10y2y	6.58	4.23	2.34	0							
10y5y	6.74	4.33	2.4	0							
10y10y	8.32	5.45	3.12	1							
10y20y	10.5	6.91	4.02	1							
10y30y	12.4	8.07	4.65	1							
20y2y	8.02	5.01	2.76	1							
20y5y	10.41	6.54	3.63	1							
20y10y	14	8.84	4.97	2							
20y20y	15.95	9.77	5.43	2							
20y30y	16.44	10.03	5.58	2							
30y2y		9.54	4.98	1							
30y5y		11.55	6.11	2							
30y10y	13.49	7.27	2.97	1.34	16.91	-1.07	-1.89	-2.9	-3.33	-3.41	
30y20y	11.13	6.05	2.49	1.13	16.64	-0.91	-1.61	-2.51	-2.92	-3.03	
30y30y	11.32	6.14	2.53	1.15	17.03	-0.94	-1.68	-2.67	-3.16	-3.36	



Smile Dynamics

⇒ Smile comoves with ATM



Delta

$$\frac{\partial}{\partial S} c(S, \sigma(S)) = \frac{\partial c}{\partial S} + \frac{\partial c}{\partial \sigma} \frac{\partial \sigma}{\partial S}$$

Delta with constant σ

Correction Term when
 σ depends on S

Hedging and risk calculation

Delta Hedges will be more stable, if the „usual“ market dynamics is captured appropriately.

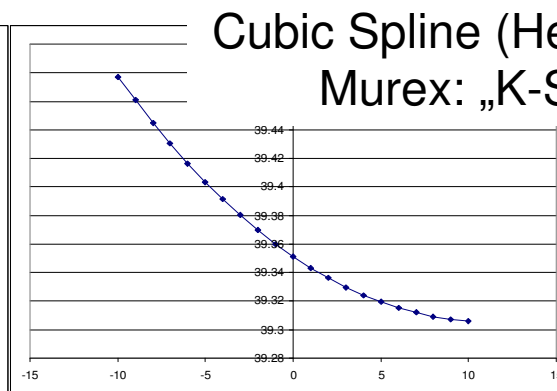
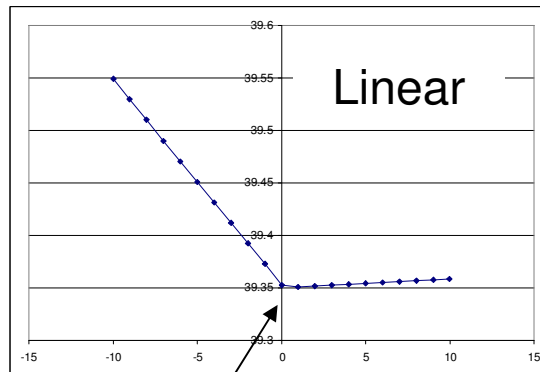
Also, risk calculation will be more suitable.

Therefore the correction term should be included both in the delta for hedging and in the delta for risk calculation.

Murex supports this for „money“ defined smiles.

Gamma

$$\frac{\partial^2}{\partial S^2} c(S, \sigma(S)) = \frac{\partial^2 c}{\partial S^2} + f\left(\frac{\partial^2 c}{\partial S \partial \sigma}, \frac{\partial^2 c}{\partial \sigma^2}, \frac{\partial c}{\partial \sigma}, \frac{\partial \sigma}{\partial S}, \frac{\partial^2 \sigma}{\partial S^2}\right)$$



Here Smile Interpolation becomes crucial

Finite Difference Derivatives can be computed, but are not stable.
Second Derivative goes to +inf if stepsize goes to 0.

Extrapolation



Outside the quoted range, one should use linear extrapolation (if not an own engineered „far strike“ - cms consistent smile anyway). Flat extrapolation produces „swings“ in the spline.

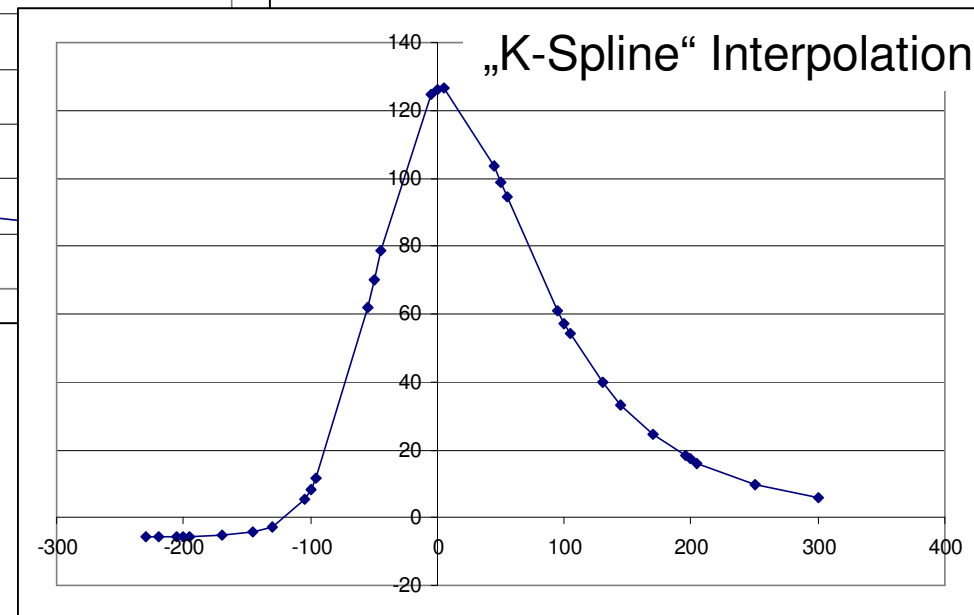
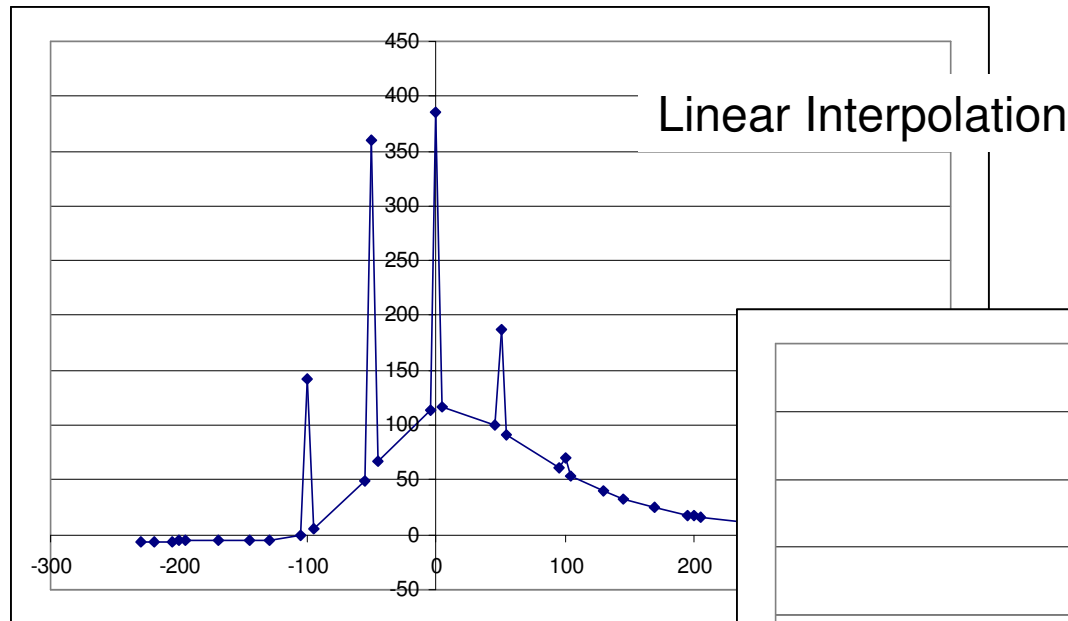
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Murex Setting:

Interpolation at bound	Extrapolate
Selected maturities	Extrapolate



Finite Difference Gammas



Murex Setting:

Parametric definition	None
Interpolation mode	K.Spline
Interpolation at bound	Linear
	K.Spline
Selected maturities	Polynomial
	Smooth

Conclusion

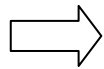
Delta and Gamma computed for hedging and risk calculation should include the correction term expressing the dependency of the volatility on the underlying.

When using finite differences for Delta and Gamma computation, it is then vital to select a smooth interpolation of the volatility smile.

In Murex, best results are obtained with „K-Spline“ interpolation and „Extrapolate at bound“.

Agenda

Gamma Computation with Smile



Cross Gammas

Bucket Deltas

5y / 7y European Payer-Swaption Deltas w.r.t. Zero Rate Buckets:

Bucket	Delta
1y	-
2y	-
3y	-
4y	-
5y	- 23,648.30
6y	991.84
7y	1,247.36
8y	1,352.71
9y	1,454.47
10y	1,525.84
11y	1,597.49
12y	41,269.11
13y	227.56
14y	-
15y	-
Total	26,018.07

5y Zero up
=> 5y / 1y Forward down
=> 5y / 7y Swap down

8y Zero up
=> 7y / 1y Forward up, 8y / 1y down
=> 5y / 7y Swap approx. same

12y Zero up
=> 11y / 1y Forward up
=> 5y / 7y Swap up



Bucket Gammas

5y / 7y European Payer-Swaption Gammas w.r.t. Zero Rate Buckets:

Gamma	5y	6y	7y	8y	9y	10y	11y	12y	13y
5y	100	- 5	- 6	- 6	- 7	- 7	- 7	- 153	- 1
6y	- 5	- 0	0	0	0	0	0	8	0
7y	- 6	0	- 1	0	0	0	0	10	0
8y	- 6	0	0	- 1	0	1	1	11	0
9y	- 7	0	0	0	- 1	1	1	12	0
10y	- 7	0	0	1	1	- 1	1	12	0
11y	- 7	0	0	1	1	1	- 1	13	0
12y	- 153	8	10	11	12	12	13	219	1
13y	- 1	0	0	0	0	0	0	1	- 0

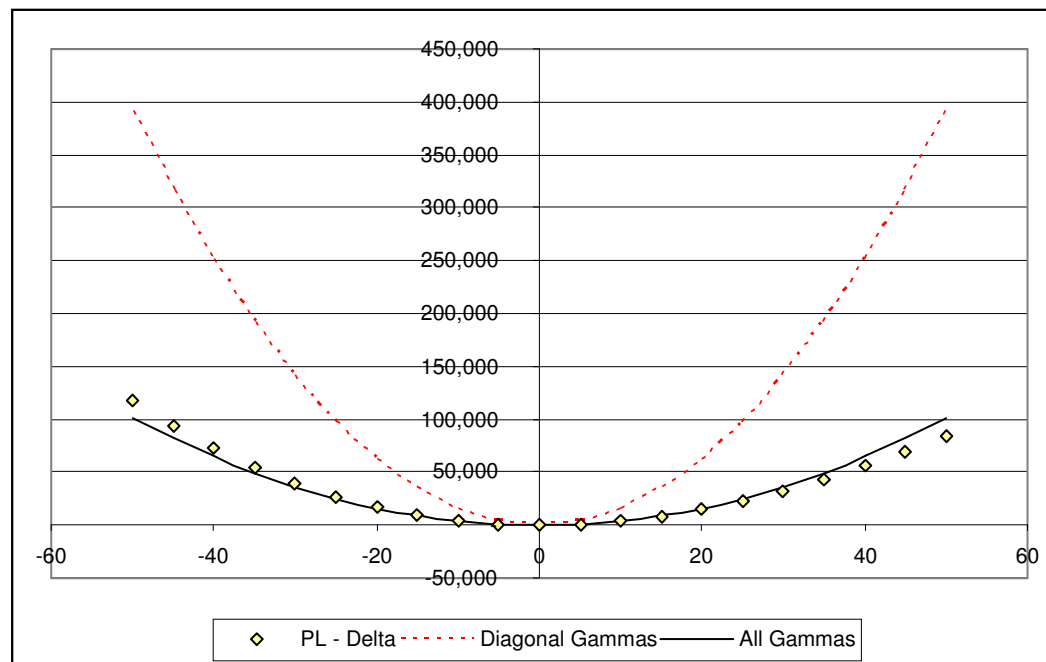
PL Explanation

Full Second Order Expansion („Delta Gamma Explanation“):

$$\Delta PL = \sum_i \frac{\partial PL}{\partial Rate_i} \Delta Rate_i + \frac{1}{2} \sum_{i,j} \frac{\partial^2 PL}{\partial Rate_i \partial Rate_j} \Delta Rate_i \Delta Rate_j$$

Problem: Limited Computational Resources might not allow the computation of the whole Gamma Matrix for all deals.

PL Explanation Diagonal vs. Full



Diagonal Gammas alone give a bad PL Estimation for parallel shifts of the rate curve

Task

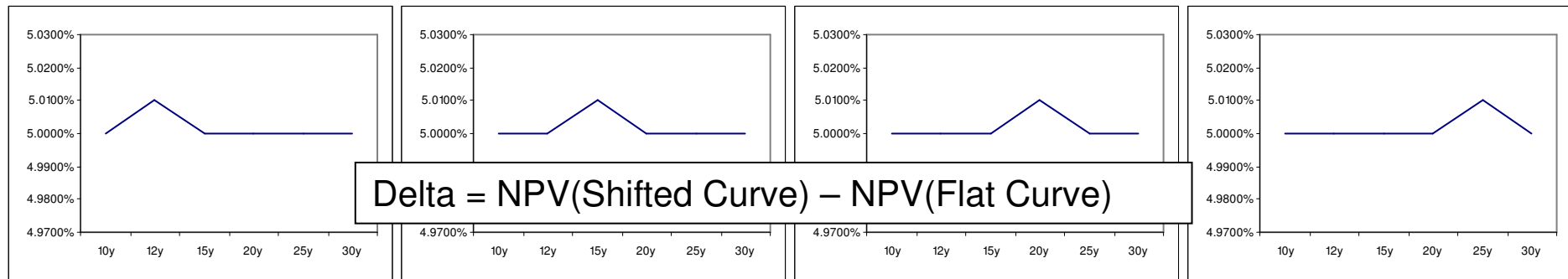
Given n Buckets, find a numerical Scheme to compute Gammas, such that

- Complexity remains $O(n)$
- A good PL Estimation at least for important subsets of Curve movements is achieved

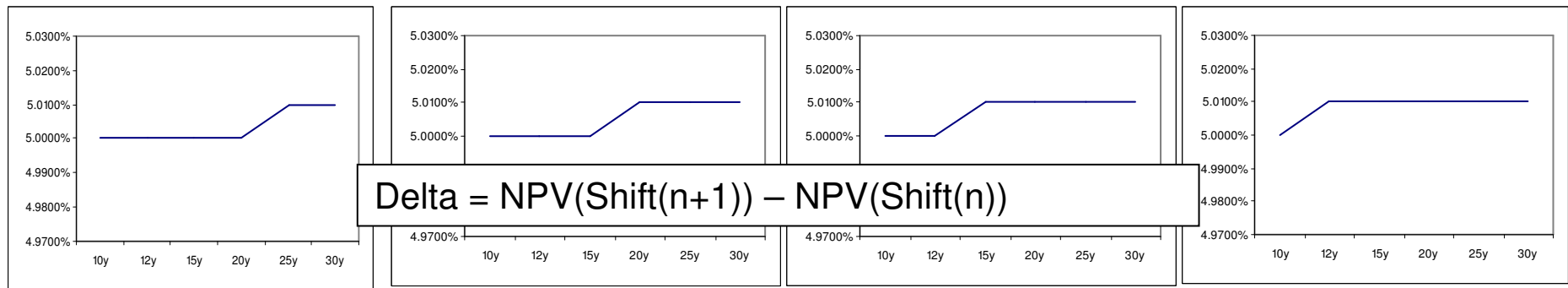
In the following we identify „Important Subsets“ with (nearly) parallel moves of the rate curve.

Approach

Instead of shifting the rate curve buckets „one by one“



... begin at the highest maturity and leave shifted buckets unchanged („backward shift“).



Approach 1 vs. 2

Shifting „one by one“:

$$\Gamma_1 = PL_{10y,10y}$$

$$\Gamma_2 = PL_{12y,12y}$$

$$\Gamma_3 = PL_{15y,15y}$$

$$\Gamma_4 = PL_{20y,20y}$$

	10y	12y	15y	20y
10y	1			
12y		2		
15y			3	
20y				4

„Backward Shift“:

$$\Gamma_1 = PL_{10y,10y} + 2 (PL_{10y,12y} + PL_{10y,15y} + PL_{10y,20y})$$

$$\Gamma_1 = PL_{12y,12y} + 2 (PL_{12y,15y} + PL_{12y,20y})$$

$$\Gamma_1 = PL_{15y,15y} + 2 PL_{15y,20y}$$

$$\Gamma_1 = PL_{20y,20y}$$

	10y	12y	15y	20y
10y	1	1	1	1
12y	1	2	2	2
15y	1	2	3	3
20y	1	2	3	4

Theoretical Foundation

Let $U \subseteq \mathbb{R}^n$ be an open subset, $f \in C^2(U)$, $x \in U$, $f(x) = 0$. Let $t_i \in \mathbb{R}^n$ represent the „backward“ shifts $t_1=(1,1,\dots,1)$, $t_2=(0,1,\dots,1,1)$, ..., $t_n=(0,0,\dots,0,1)$ and $t_{n+1} = 0$. Then the following relations hold for all $i = 1, \dots, n$:

$$\lim_{h \rightarrow 0} \frac{(f(x + ht_i) - f(x + ht_{i+1})) - (f(x - ht_i) - f(x - ht_{i+1}))}{2h} = \frac{\partial f}{\partial x_i}(x)$$

$$\lim_{h \rightarrow 0} \frac{(f(x + ht_i) - f(x + ht_{i+1})) + (f(x - ht_i) - f(x - ht_{i+1}))}{h^2} = \frac{\partial^2 f}{\partial x_i^2}(x) + \sum_{j=1}^{n-i} \left(\frac{\partial^2 f}{\partial x_i \partial x_{i+j}}(x) + \frac{\partial^2 f}{\partial x_{i+j} \partial x_i}(x) \right)$$

Example: CMS Spread Floor

CMS Spread Floor at 0%, 10y

Maturity	Delta old	Delta new	Gamma old	Gamma new
1y	-175	-175	3	-6
2y	-596	-596	17	-26
3y	-1,833	-1,833	91	-68
4y	-2,539	-2,540	198	-105
5y	-1,064	-1,065	516	-87
7y	1,946	1,946	898	-147
10y	28,472	28,472	991	45
15y	-18,568	-18,568	314	383
30y	-2,738	-2,738	12	12
Total	2,906	2,904	3,039	1

PL Estimation Error for parallel rate curve shifts:

	-60	-30	0	30	60
old	546.94	136.74	-	136.76	547.04
new	0.09	0.03	-	0.04	0.17

(shifts and errors in basispoints of reference notional)

Real life examples

The new backward shift method is – of course – not always better than the old method.

Here are Delta Gamma PL Estimations for real rate curve movements and real deals:

PL 12.06. to	PL	Delta Gamma old	Delta Gamma new	Error old	Error new
25.06.	-112,369.15	-105,287.22	-110,405.14	7,081.93	1,964.01
24.06.	-78,775.85	-73,667.02	-76,888.15	5,108.84	1,887.70
23.06.	-64,707.09	-63,474.97	-65,578.34	1,232.12	-871.25
15.06.	-74,327.82	-72,785.35	-74,129.31	1,542.47	198.51

New method is better than old

PL 12.06. to	PL	Delta Gamma old	Delta Gamma new	Error old	Error new
25.06.	-9,017,552.58	-8,457,076.56	-8,374,980.11	560,476.02	642,572.47
24.06.	-7,011,072.48	-6,567,190.27	-6,505,016.19	443,882.21	506,056.29
23.06.	-5,854,888.71	-5,477,069.85	-5,437,135.71	377,818.86	417,752.99
15.06.	-4,481,389.50	-4,322,164.71	-4,302,556.99	159,224.79	178,832.51

New method is worse than old

Conclusion

The new method may be better, but also worse than the old method. This depends on the deal (its gamma matrix) and how the rate curve movement fits (are significant cross gammas multiplied by an approximately correct shift?).

However, for (nearly) parallel rate curve movements, the new method is always better than the old.

Believing in the „general result“ that the first principal component of rate curve movements is nearly a parallel shift, the new method is preferable in this statistical sense.