Operational Risk: Which goodness-of-fit test?

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Summary

Operational Risk in Banking

- **Operational risk is categorised according to a matrix of 8 business lines and 7 risk types.**

- **3 possible approaches:**
  - Basic Indicator Approach: percentage of gross income
  - Standardised Approach: summation of percentages of gross income Business lines
  - Advanced Measurement Approach: LDA

- **2008: first calculation with LDA**

LDA: an advanced method (1)

- **LDA is an actuarial method adapted to operational risk.**

- **Capital requirement = $\text{VaR}_{99,9\%}$ of the annual loss**
  - To back test this VaR, we need at least 1000 years of data.
  - We only use 5 years of losses (above a threshold).

- **The annual loss distribution is modelised using two dimensions:**
  - Severity: amounts of losses
  - Frequency: annual number of losses
  - $Annual \ \ loss = \sum_{i=0}^{nb.losses} X_i$

- **We can only judge the severity distribution fit.**

LDA: an advanced method (2)

- **Step 1: severity estimation**
  Lognormal with parameters µ et λ

- **Step 2: frequency estimation**
  Poisson with parameter λ

- **Step 3: Monte Carlo simulation**

1
2
¦
k $\longrightarrow$ $\boldsymbol{Nb.}$ $\longrightarrow$ $L = \sum_{n=1}^{Nb.} Loss_n$
¦
N

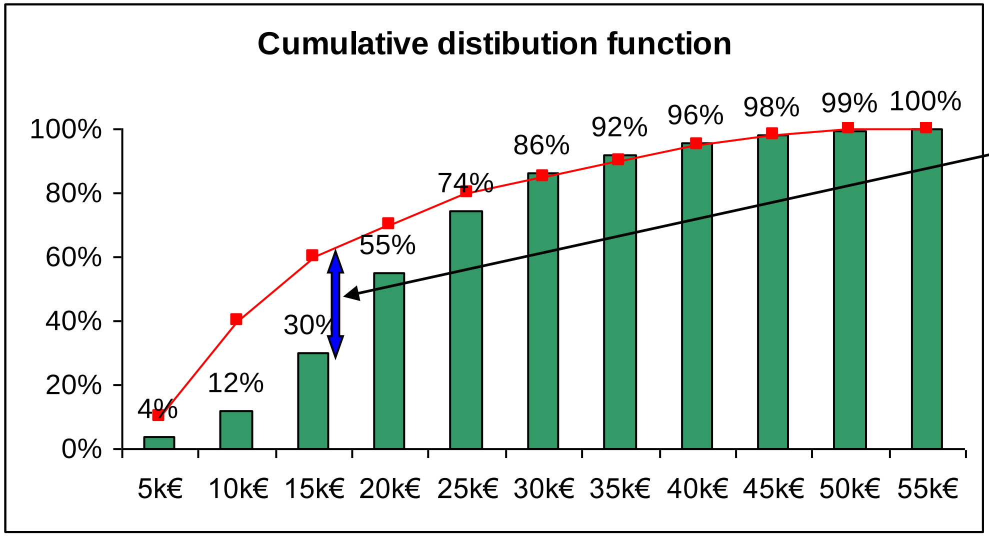
VaR = 99,9 % quantile

N = 5 000 000

Goodness-of-fit tests (1)

- **The goal of these tests is to check that the theoretical severity distribution and the empirical one are close enough.**

- **More precisely, the tests are a distance measure between the two cumulative distributions.**



Gap between the theoretical distribution and the empirical one

Goodness-of-fit tests (2)

Kolmogorov-Smirnov's tests: LOCAL TESTS

Empirical cdf: $F_n(x)$ — Theoretical cdf: $\hat{F}(x)$

$$KS = \max \Psi(\hat{F}(x)) \left| F_n(x) - \hat{F}(x) \right|$$

- $\Psi(\hat{F}(x)) = 1$ — Kolmogorov-Smirnov
- $\Psi(\hat{F}(x)) = \left\{1 - \hat{F}(x)\right\}^{-1}$ — Modified Kolmogorov-Smirnov

Cramer von Mises' tests: QUADRATIC TESTS (GLOBAL)

$$Q = n \int_{-\infty}^{+\infty} (F_n(x) - \hat{F}(x))^2 \Psi(\hat{F}(x)) d\hat{F}(x)$$

- $\Psi(\hat{F}(x)) = 1$ — Cramer von Mises
- $\Psi(\hat{F}(x)) = \left\{\hat{F}(x)(1 - \hat{F}(x))\right\}^{-1}$ — Anderson Darling
- $\Psi(\hat{F}(x)) = \left\{1 - \hat{F}(x)\right\}^{-2}$ — Modified Anderson Darling

Goodness-of-fit tests (3)

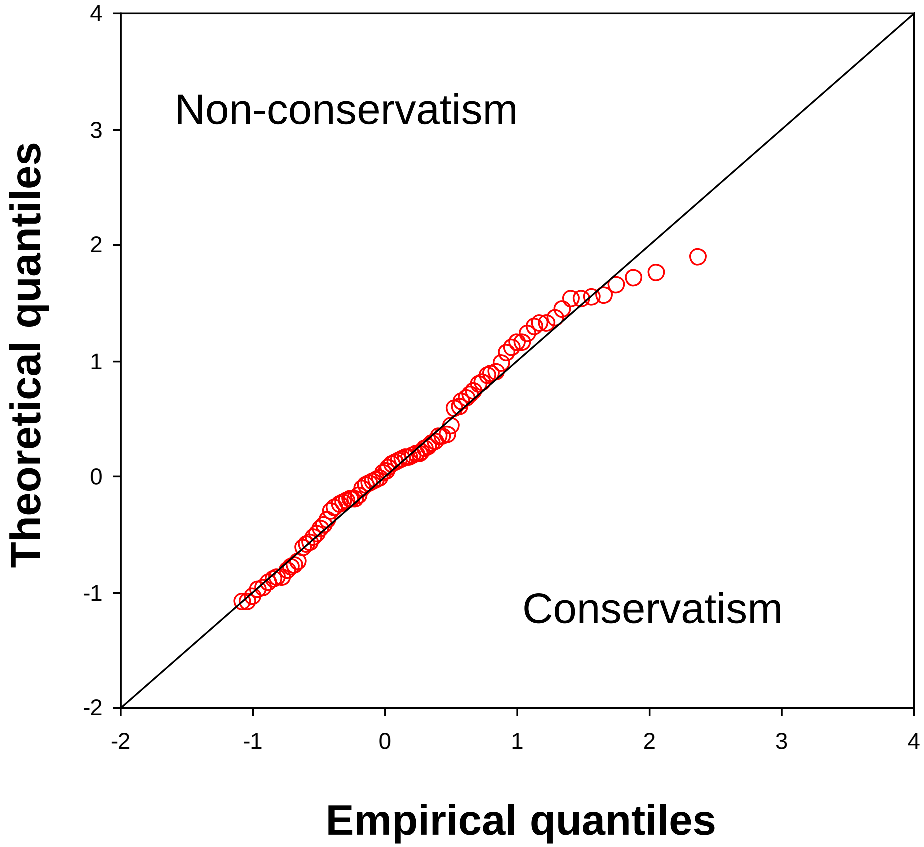
- **Then, we will focus on the three following tests:**
  - KS: a very common test but particularly sensitive to extreme values
  - CVM: a good one to judge the global fit
  - AD_up: often presented as the best regarding operational risk.

- **To better fit the annual loss distribution, we have to be more accurate for the higher losses.**
  - Ex: Annual loss = 100 + 1 000 000

- **This fact is accentuated by the high value of the VaR’s quantile: 99,9%.**

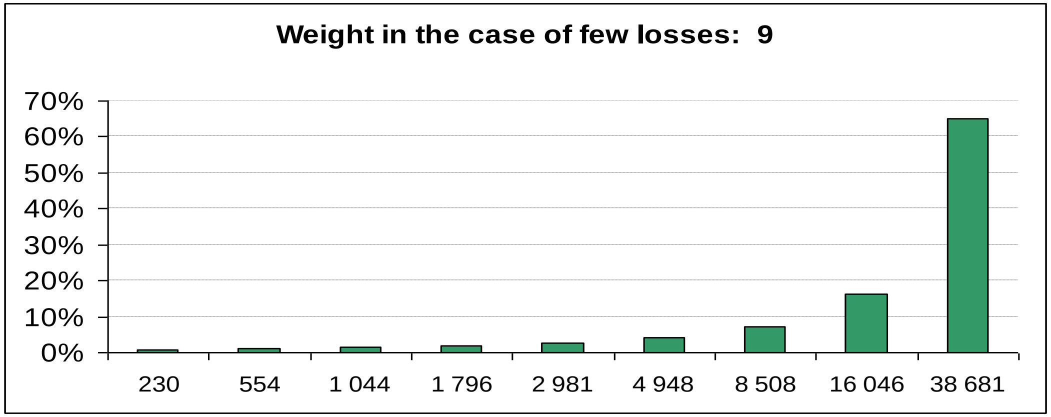
Goodness-of-fit tests (4)

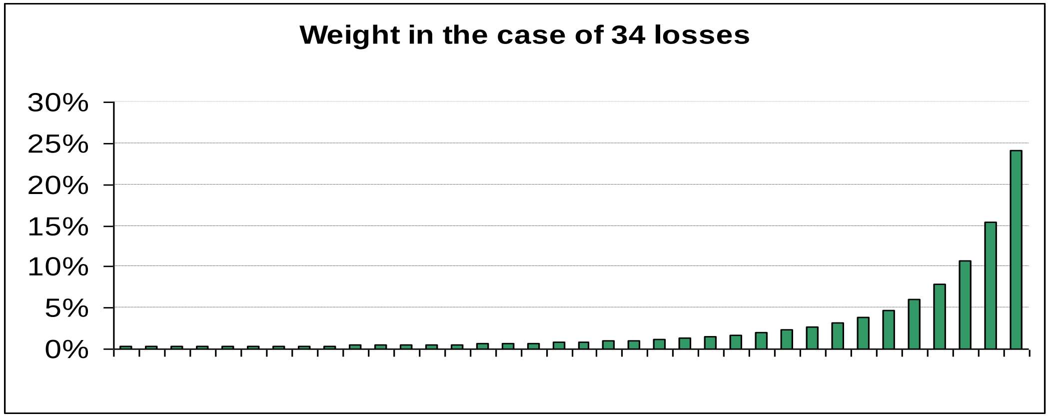
- Q-Q plot: graphical illustration of fit



How to judge GOF tests? (1)

- **Is AD_up always the best choice?**





With few losses, AD_up gives a very consequent weight to the highest loss.

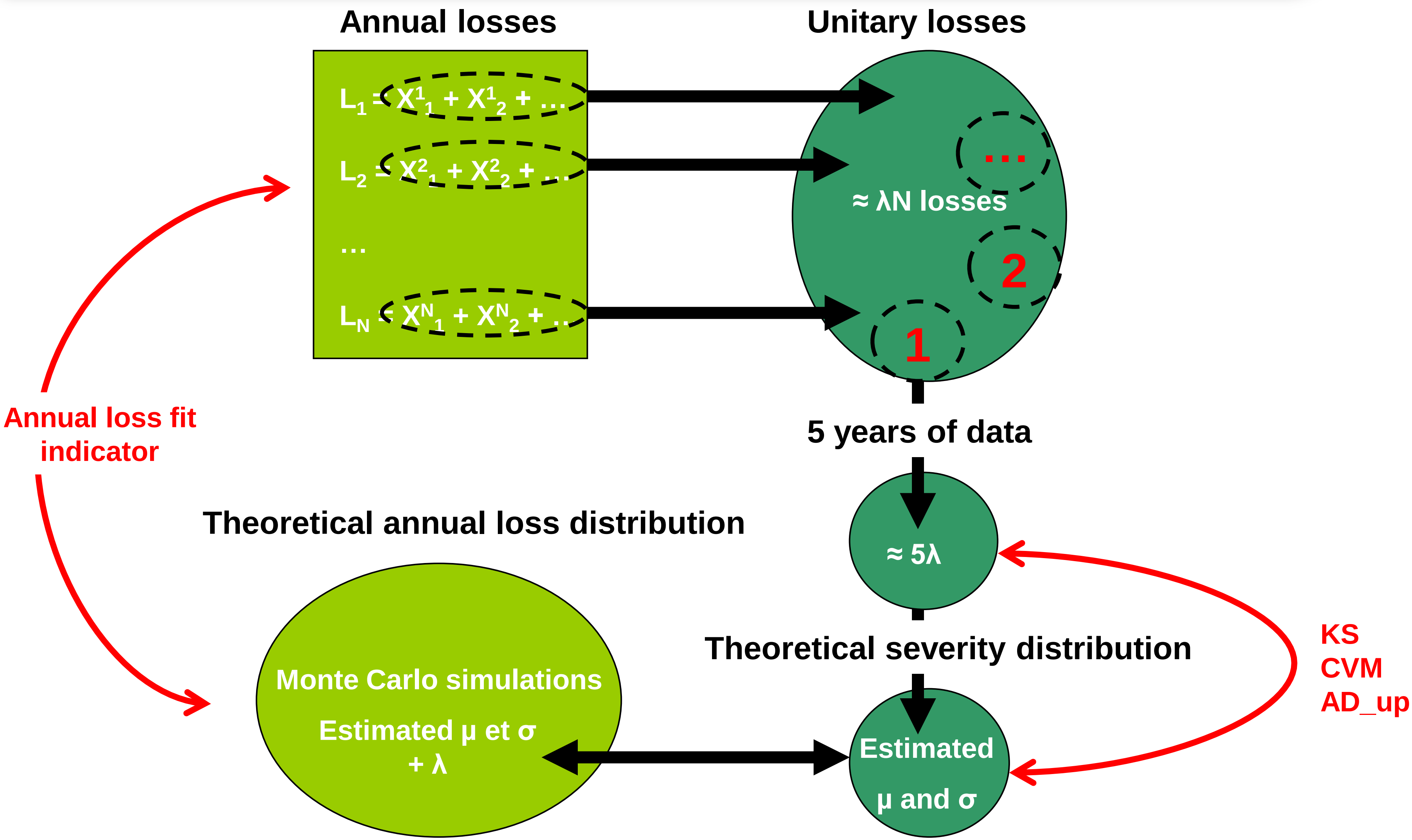
How to judge GOF tests? (2)

- With only 5 years of data, we can't observe which severity test implies the best fit on the annual loss distribution.

- To avoid this problem, we establish some tests with sample data:
  - $X_i \sim LN(8, 2)$
  - $p_i \sim \mathcal{P}(\lambda)$

- The idea is to determine which test has the highest correlation with an annual loss fit indicator.

How to judge GOF tests? (3)



How to judge GOF tests? (4)

	KS	CVM	AD_up	Annual loss fit indicator
1	0,279	0,177	8,079	0,652
2	0,365	0,025	4,236	0,895
3	0,493	0,241	6,001	1,023
…	…	…	…	…
Ncorr	0,125	0,663	2,369	0,941

Rank transformation

	KS	CVM	AD_up	Annual loss fit indicator
1	2 478	1 002	7 412	563
2	3 780	125	932	1 245
3	4 569	3 610	2 014	6 541
…	…	…	…	…
Ncorr	369	7 489	145	3 200

We calculate the correlations between the GOF tests and the annual loss fit indicator.

How to judge GOF tests? (5)

- **We establish 2 annual loss fit indicators:**
  - **1) CDF indicator**

    It's a discrete CVM test, i.e. the sum of all the gaps:

    $$CDF\ indicator = \sum_{i=1}^{N} \left| F_N(L_i) - \hat{F}(L_i) \right|$$

    Is the annual loss distribution globally well fitted?

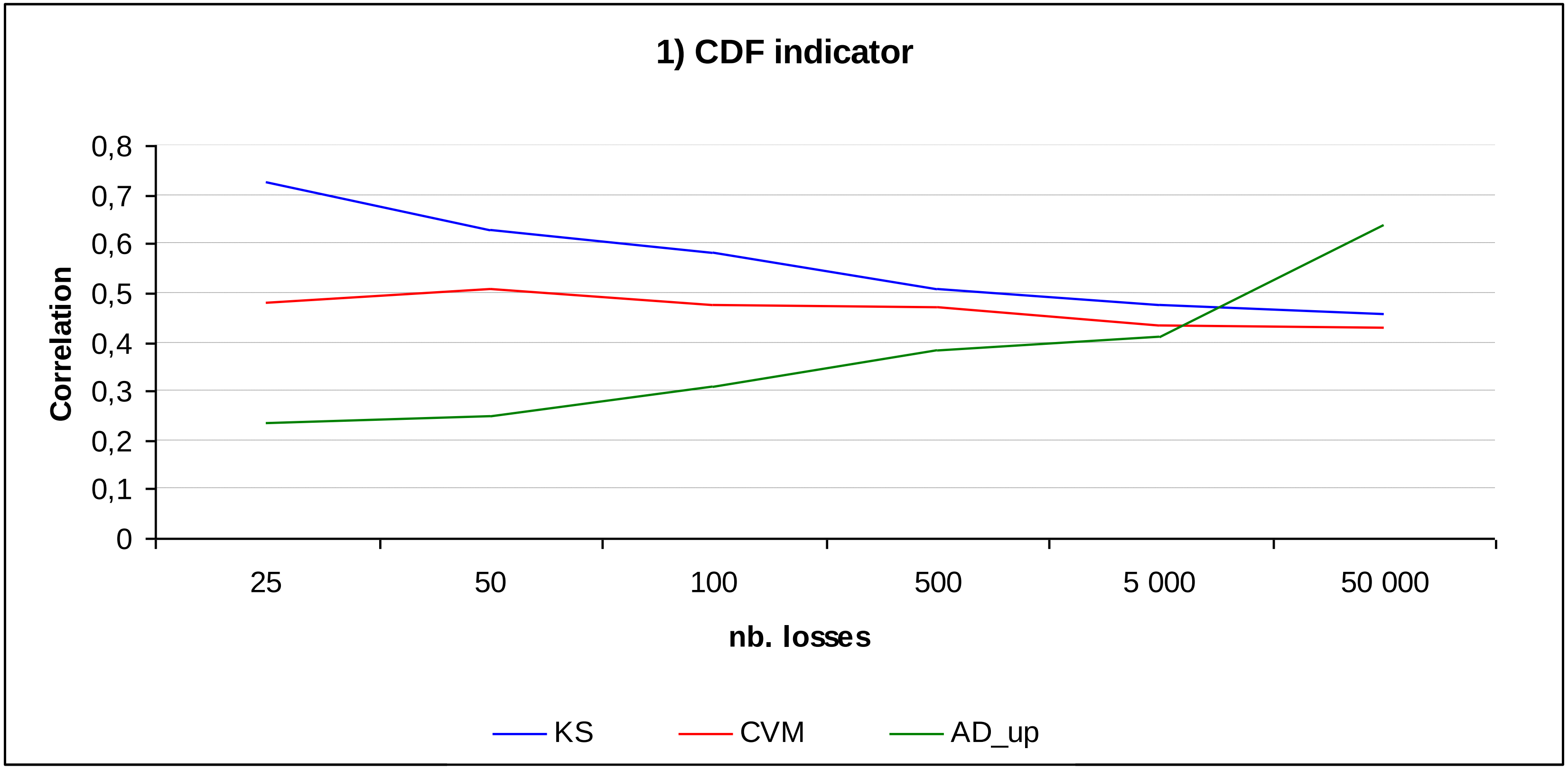
  - **2) VaR indicator**

    It corresponds to the difference between the empirical VaR and the theoretical one:

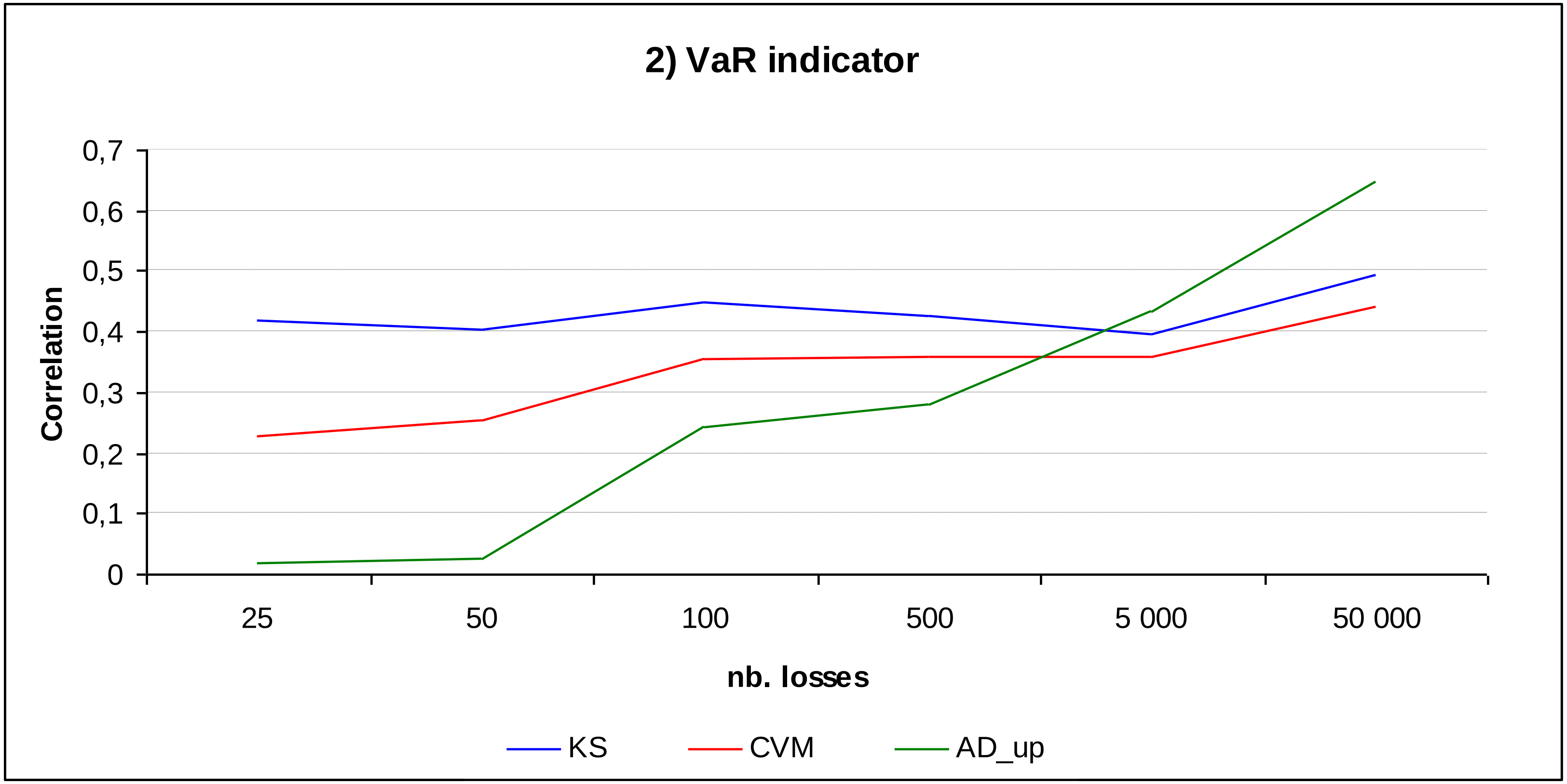
    $$VaR\ indicator = \frac{\left| VaR_{emp} - VaR_{th} \right|}{VaR_{emp}}$$

    We choose the 90% quantile instead of 99,9% to accelerate the simulations.

Results: comparison of usual GOF tests (1)



Results: comparison of usual GOF tests (2)



Results: comparison of usual GOF tests (3)

- AD_up is a good choice when nb. Losses > 500: it's not often the case for operational risk.
- KS appears to have good results but they are certainly biased by the sample context.
- These simulations could and should be completed by:
  - Robust tests
  - Sampling data with mixture of laws
  - Bootstrapping real losses
  - …

New GOF tests (1)

- **Other tests could be used. Here, we suggest two of them:**
  - A test that gives a weight equal to the distance between the theoretical quantiles and empirical ones:

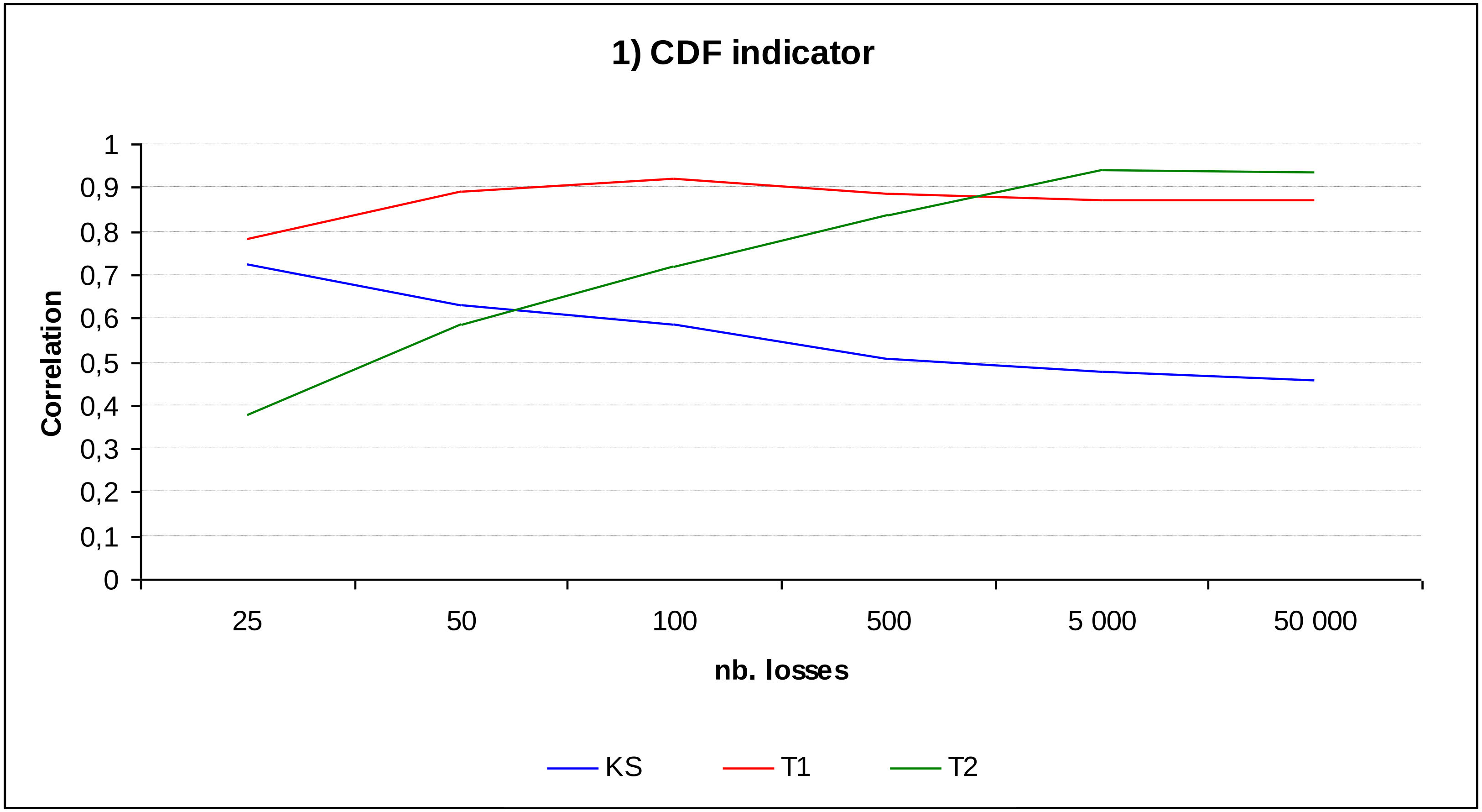
$$T1 = \sum_{i=1}^{nb.losses} \left| \left[X_i - F_N^{-1}(\hat{F}(X_i)) \right] \times \left[F_N(X_i) - \hat{F}(X_i) \right] \right|$$

  - A test equal to the sum of these weights:

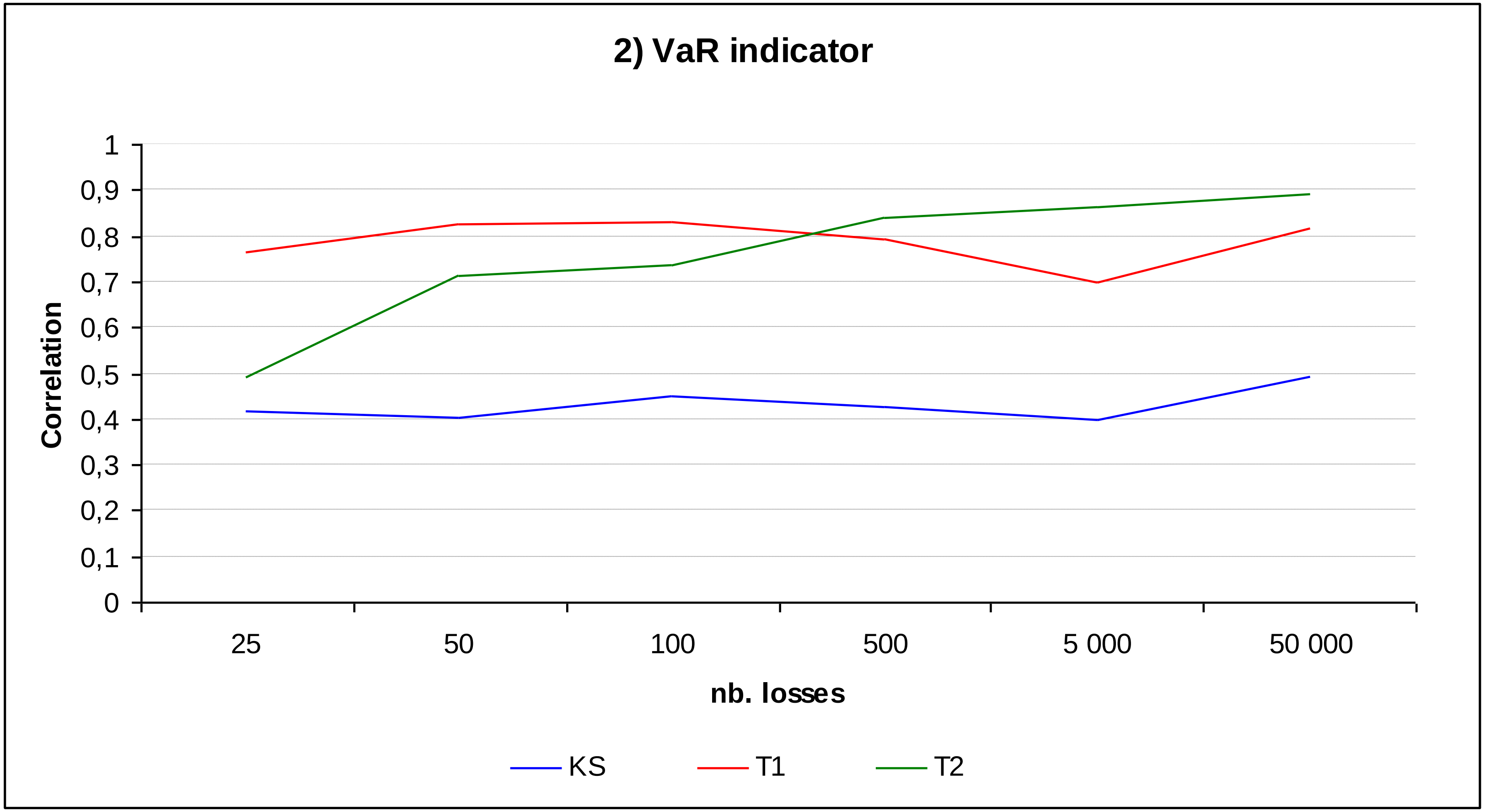
$$T2 = \sum_{i=1}^{nb.losses} \left| X_i - F_N^{-1}(\hat{F}(X_i)) \right|$$

- **These tests are discrete.**

New GOF tests (2)



New GOF tests (3)



Conclusion

- **Judging the quality of goodness-of-fit test for Operational risk is a complicated task due to some particularities:**
  - Few losses to fit and test the estimation
  - Difficulties to backtest the results due to the high VaR quantile

- **AD_up is usually used with a lot of observations. It may give too much weight to the highest losses, which is not suitable for annual loss estimation.**

- **This study does not affirm that a goodness-of-fit is better than an other one but that we have to be really careful with the particularities of operational risk.**