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MARSH MERCER KROLL
GUY CARPENTER OLIVER WYMAN

# Wavelet Analysis of Business Cycles
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Dr. M.J. van der Burgt

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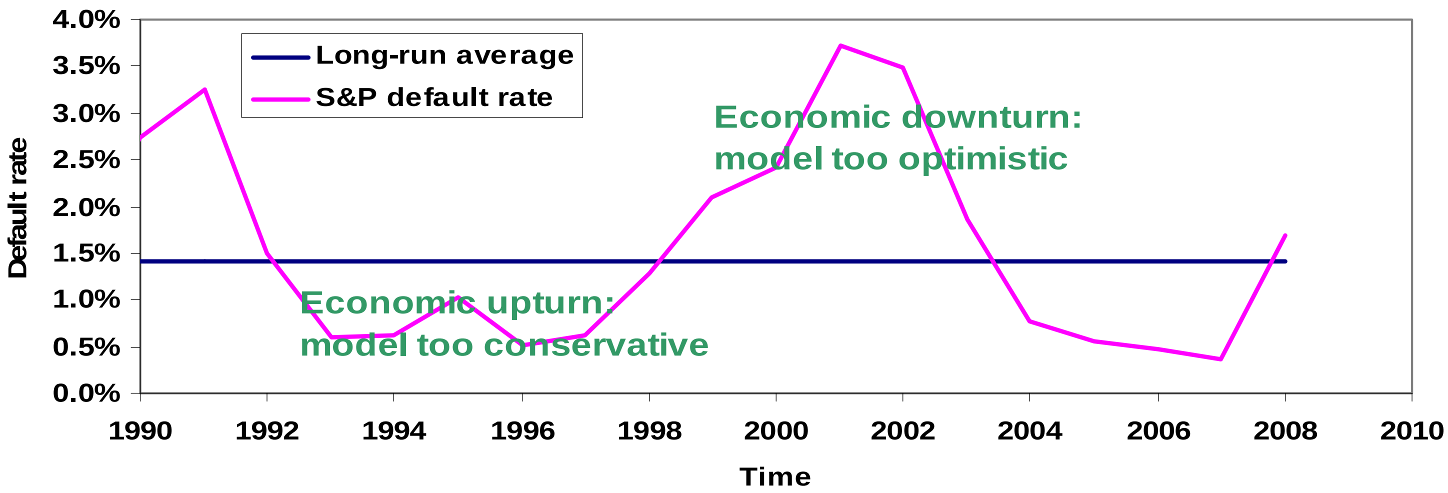
# Agenda

- Introduction: why study business cycles?
- Fourier analysis of business cycles.
- What are wavelets?
- Wavelet analysis of business cycles.
- Multi-Resolution Analysis.
- Other applications of this study: pension funds.
- Conclusions.

# Introduction: why study business cycles?

- Credit risk modelling: estimation of the Probability of Default (PD) for each rating class.
- Basel II paragraph 447: “PD must be a long-run average of one-year default rates for borrowers in the grade”. Long-run is assumed to be the length of (at least) one business cycle.
- Consequences in validation of PD rating models:

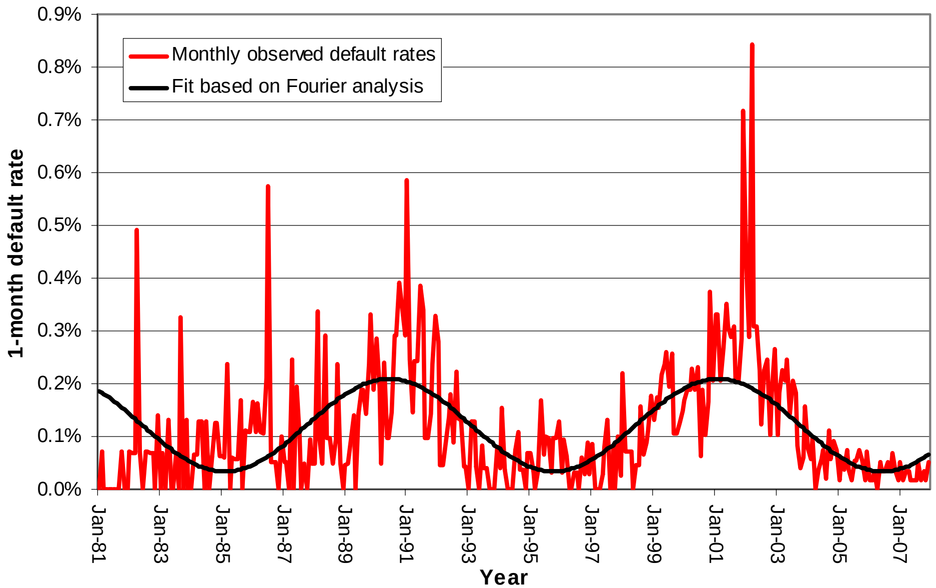


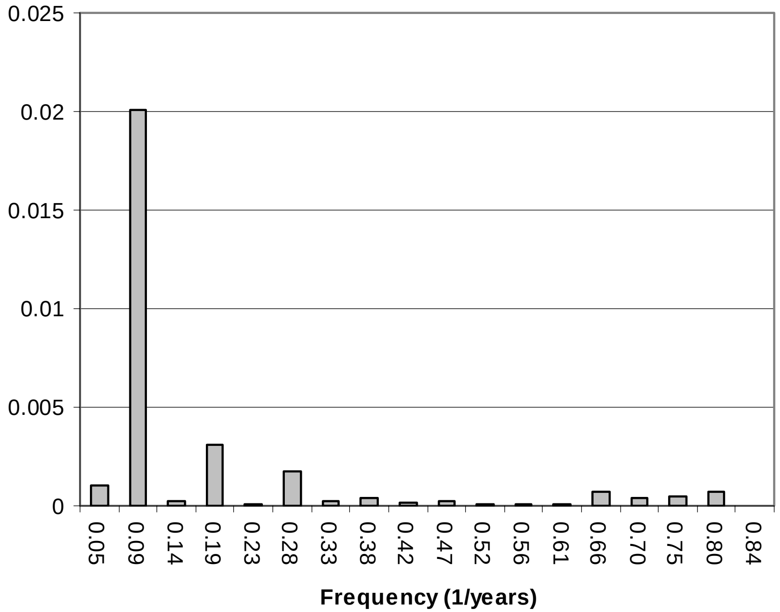
# Introduction: why study business cycles?

- Two questions are relevant in credit risk modelling and validation:
  – How long is the business cycle?
  – Where exactly are we in the business cycle?
- To be answered by:
  – Fourier Analysis.
  – Wavelet Analysis.
- Used data: monthly S&P default rates over period 1981 – 2007.

# Fourier analysis of business cycles.

- Overlap between time-series and a harmonic function (sine or cosine).
- Plot the squared overlap as a function of frequency: transformation from time space to frequency space:
  – Length of business cycle: 10.67 years;
  – Localization in time is lost.



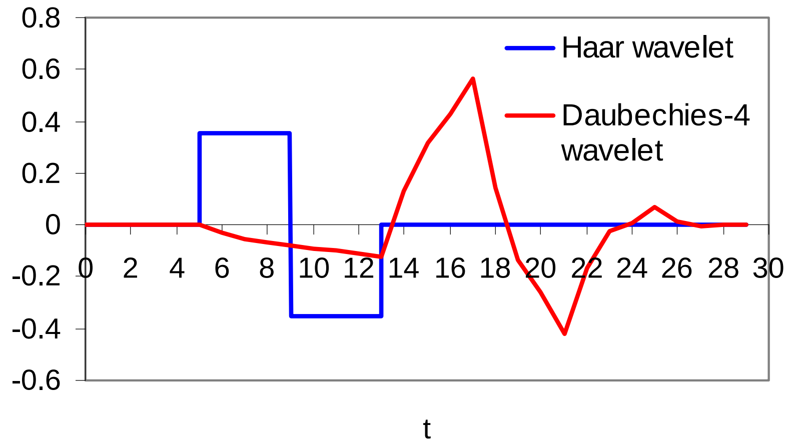



# What are wavelets?

- Properties of Wavelets Ψ:

$$\int_{-\infty}^{\infty} \psi(x)dx = 0 \qquad \int_{-\infty}^{\infty} [\psi(x)]^2 dx = 1$$

- Examples: Haar, Daubechies-4




- Wavelets can be dilated by τ and translated by t:

$$\psi_{\tau,t}(x) = \frac{1}{\sqrt{\tau}} \psi\left(\frac{x-t}{\tau}\right)$$

- Allow transformation from time space to time-frequency space.
- Applications of wavelets [Lepik2005, Percival2000] : time series analysis, solving differential equations, image processing, data compression.

## What are wavelets?

Continuous Wavelet Transformation (CWT)

CWT is the overlap between a function f(x) and a wavelet:

$$W(\tau,t)=\int_{-\infty}^{\infty} f(x)\cdot\psi_{\tau,t}(x)dx=\langle f,\psi_{\tau,t}\rangle$$

Discrete Wavelet Transformation (DWT)

DWT by standardizing to a dyadic scale: $\tau_j = 2^j$:

– Father wavelet or scaling function: $\varphi_{j,t}(x)=\frac{1}{2^{j/2}}\varphi\left(\frac{x-t}{2^j}\right)$

– Mother wavelet: $\psi_{j,t}(x)=\frac{1}{2^{j/2}}\psi\left(\frac{x-t}{2^j}\right)$

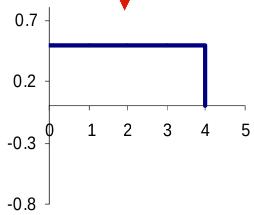
The higher j (“resolution”), the smoother the time-series.

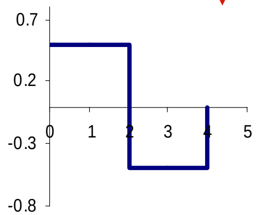
# What are wavelets?

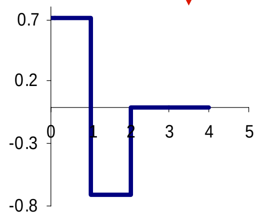
- DWT: an example:

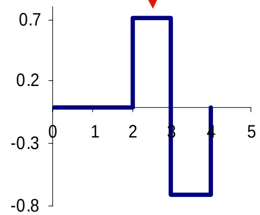
$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{x_0 + x_1 + x_2 + x_3}{2} \cdot \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \frac{x_0 + x_1 - x_2 - x_3}{2} \cdot \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} + \frac{x_0 - x_1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} + \frac{x_2 - x_3}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

$$= V_{2,0} \cdot \varphi^H_{2,0} + W_{2,0} \cdot \psi^H_{2,0} + W_{1,0} \cdot \psi^H_{1,0} + W_{1,2} \cdot \psi^H_{1,2}$$




- DWT corresponds to averaging (low-pass filter) and differencing (high-pass filter)
- Due to orthogonality, the number of observations must be a power of two.

## What are wavelets?

- Improvement of DWT: Maximum Overlap Discrete Wavelet Transform (MODWT), in which wavelet coefficients are calculated by recursion:

$$V_j = \hat{A}_j V_{j-1}$$

$$W_j = \hat{B}_j V_{j-1}$$

- where $\mathbf{V_j = [\ldots, V_{j,t}, \ldots]^T}$ with $\mathbf{V_{0,t} = X_t}$, and $\mathbf{W_j = [\ldots, W_{j,t}, \ldots]^T}$. Matrices $A_j$ and $B_j$: see paper for technical details [Burgt2009].

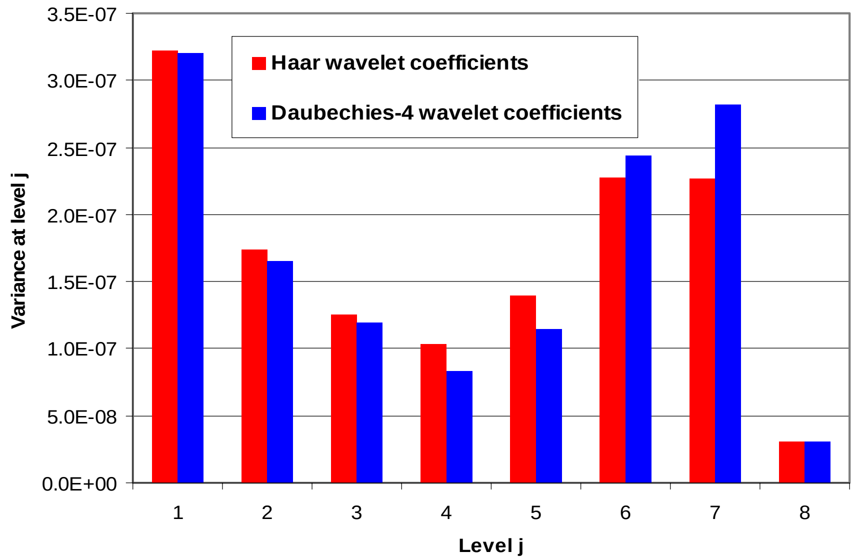
- Analyse variance at different resolution levels j (or frequencies $2^{-j}$):

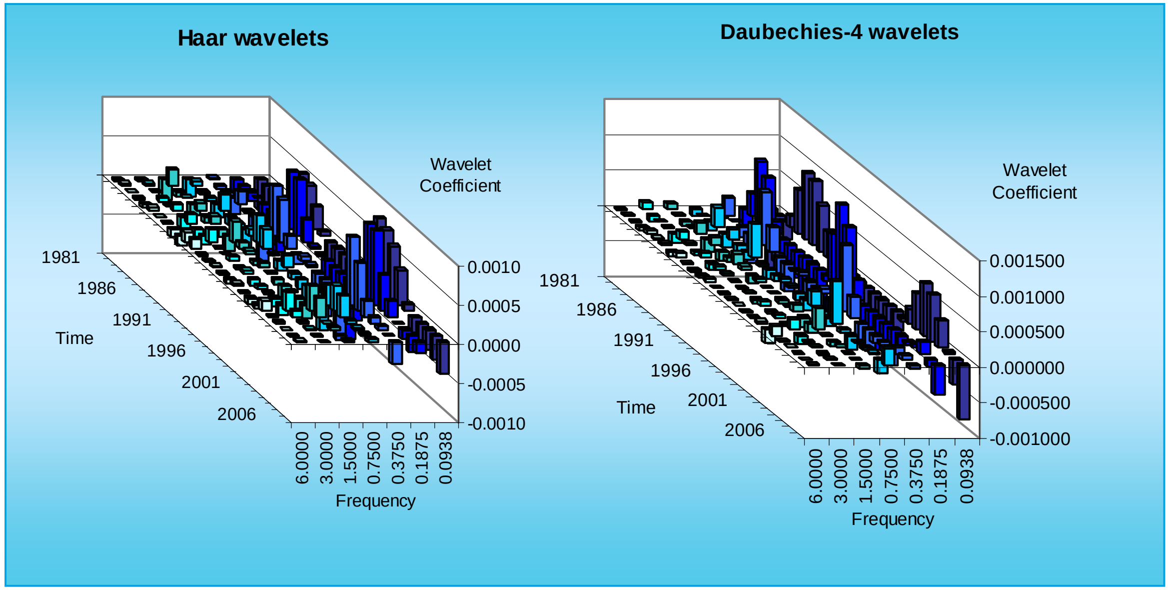
$$\sigma_X^2 = \sum_{j=1}^{J} \left( \frac{1}{N} \left| W_j \right|^2 \right) + \left( \frac{1}{N} \left| V_J \right|^2 - \left\langle X \right\rangle^2 \right)$$

- The dominant wavelet is identified by the largest wavelet coefficient $W_{j,t}$.

# Wavelet analysis of business cycles.

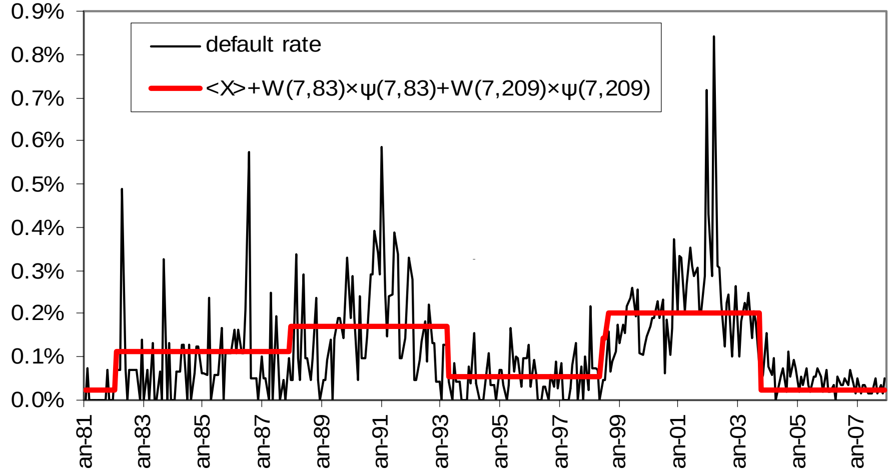
- Variance decomposed at different levels j;
  - Component j = 1 is attributed to noise;
  - Component j = 7: period of $2^7$ months (10.67 years);
- Analysis of $W_{j,t}$ at j = 7 reveals two dominant wavelets:
  - At j = 7: t = 83 and t = 209 (Haar wavelets).
  - At j = 7: t = 63 and t = 199 (Daubechies-4 wavelets).

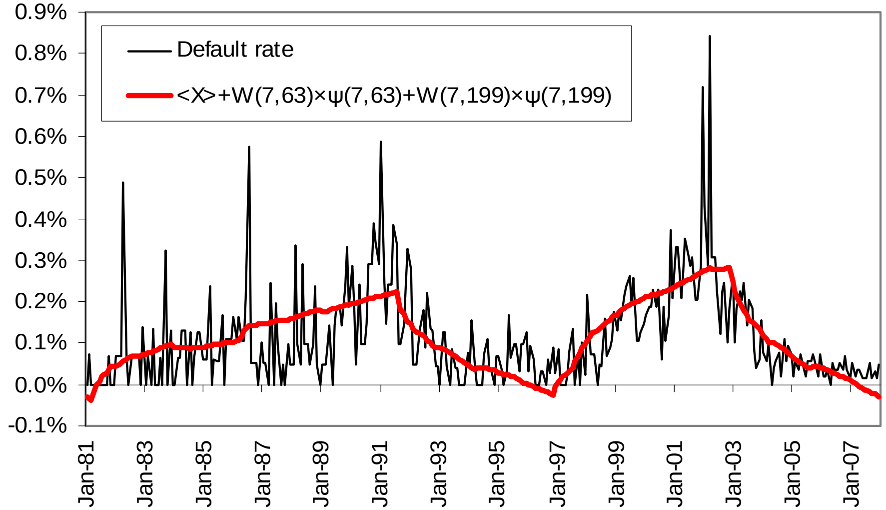


# Wavelet analysis of business cycles.

- Two business cycles of 10,67 years:
  - Starting with downturn at December 1987 and June 1998 (Haar wavelets).
  - Starting with downturn at April 1986 and August 1997 (Daubechies-4 wavelets).
- Due to “blocky” nature of the Haar wavelets, accuracy is low and Daubechies-4 wavelets give better accuracy than Haar wavelets.



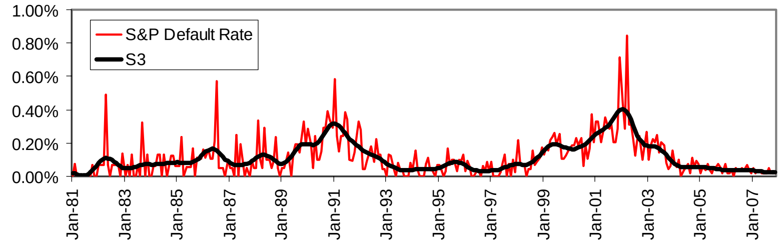



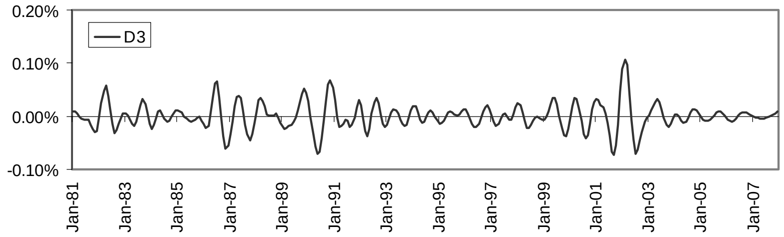
# Multi-Resolution Analysis.

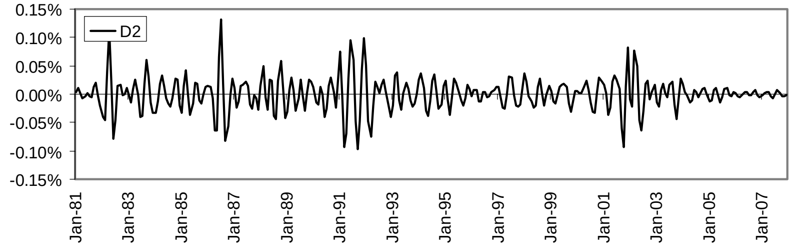
- Multi-Resolution Analysis (MRA): smoothed time-series $S_J$ and j-level details $D_j$ are calculated from X, $A_j$ and $B_j$ :

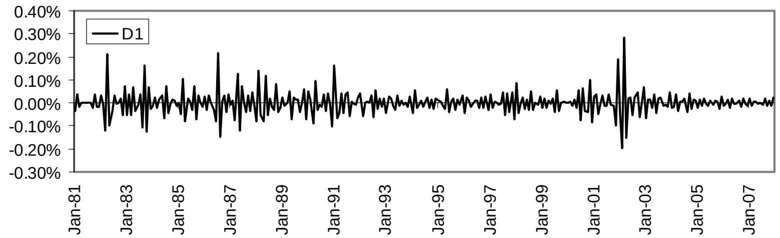
$$X = \sum_{j=1}^{J} D_j + S_J$$

- S&P default rate = S3 + D3 + D2 + D1 using Daubechies-4 wavelets.
- Application: denoising.
- Analysis of S3 rather than S&P default rate leads to similar results.



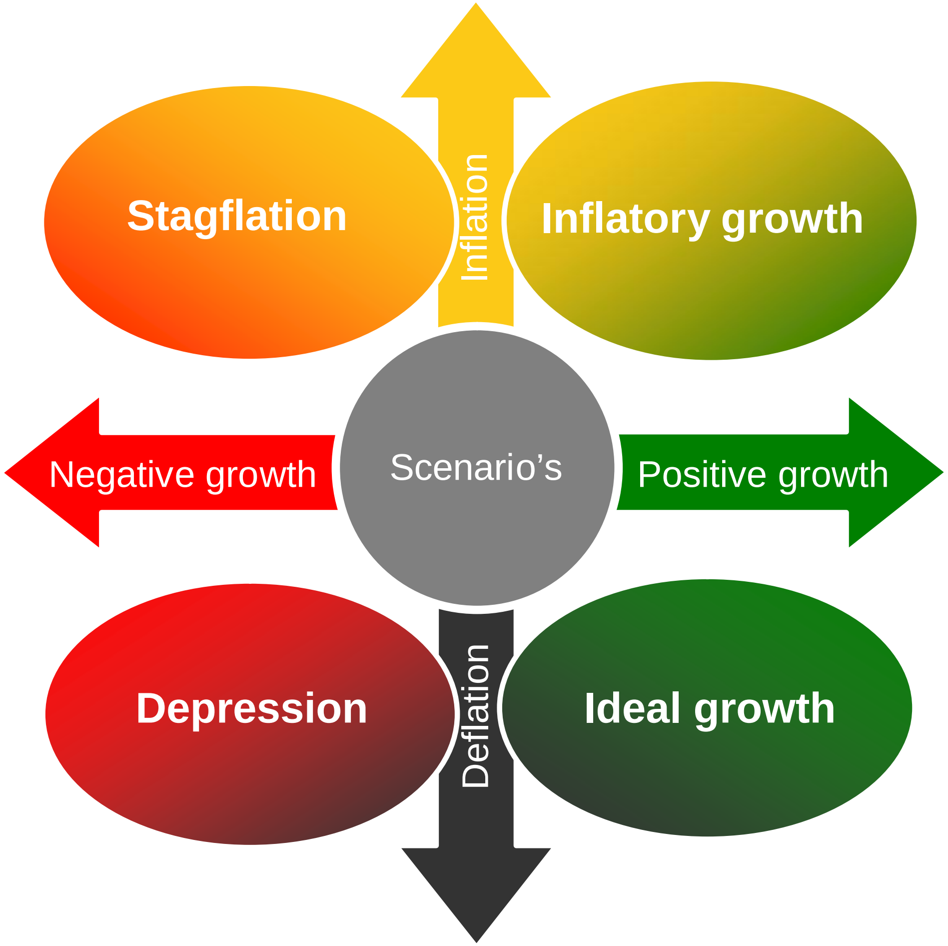
# Other applications of this study: pension funds.

| | S&P annual default rate | PME funding level | ABP funding level | Pf ABN Amro funding level |
|---|---|---|---|---|
| 2008 | 1.69% | 88% | 90% | 106% |
| 2007 | 0.36% | 135% | 140% | 139% |
| 2006 | 0.46% | 128% | 134% | 131% |
| 2005 | 0.57% | 123% | 120% | 116% |
| 2004 | 0.77% | 116% | 121% | 118% |
| | Correlation | -0.993 | -0.967 | -0.834 |

- Pension funds are sensitive to business cycles: in 2009, about 350 of the 650 Dutch pension funds were underfunded.
- ALM for Pension Funds: long-term policy (15 years) with respect to Premium, Indexation and Investments.

# Other applications of this study: pension funds.

- ALM: How solvent is a Pension Fund under different scenario's?
- Scenario's characterised by:
  - Business cycle (growth / recession).
  - Other cycle (inflation / deflation).
- ALM: extend wavelet analysis to two dimensions: for growth and inflation.



# Conclusions.

- Fourier analysis: a transformation from time space to frequency space.
- Wavelet analysis:a transformation from time space to time-frequency space.
- Two business cycles with a length of 10.67 years identified, starting at April 1986 and August 1997 with a downturn respectively.
- Business cycle analysis is relevant for:
  – Credit risk management.
  – Policy design for pension funds.
- Questions?

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MARSH MERCER KROLL
GUY CARPENTER OLIVER WYMAN

**Dr. M.J. van der Burgt**, ALM Consultant
**Mercer** | Retirement - Investment Consulting

Startbaan 6, 1185 XR Amstelveen
Postbus 2271 1180 EG Amstelveen
+31 20 4313957 | Fax +31 20 4313789
Marco.vanderBurgt@mercer.com