Frailty models for defaults:

decomposing defaults into firm-specific, industry, macro, and frailty credit risk factors

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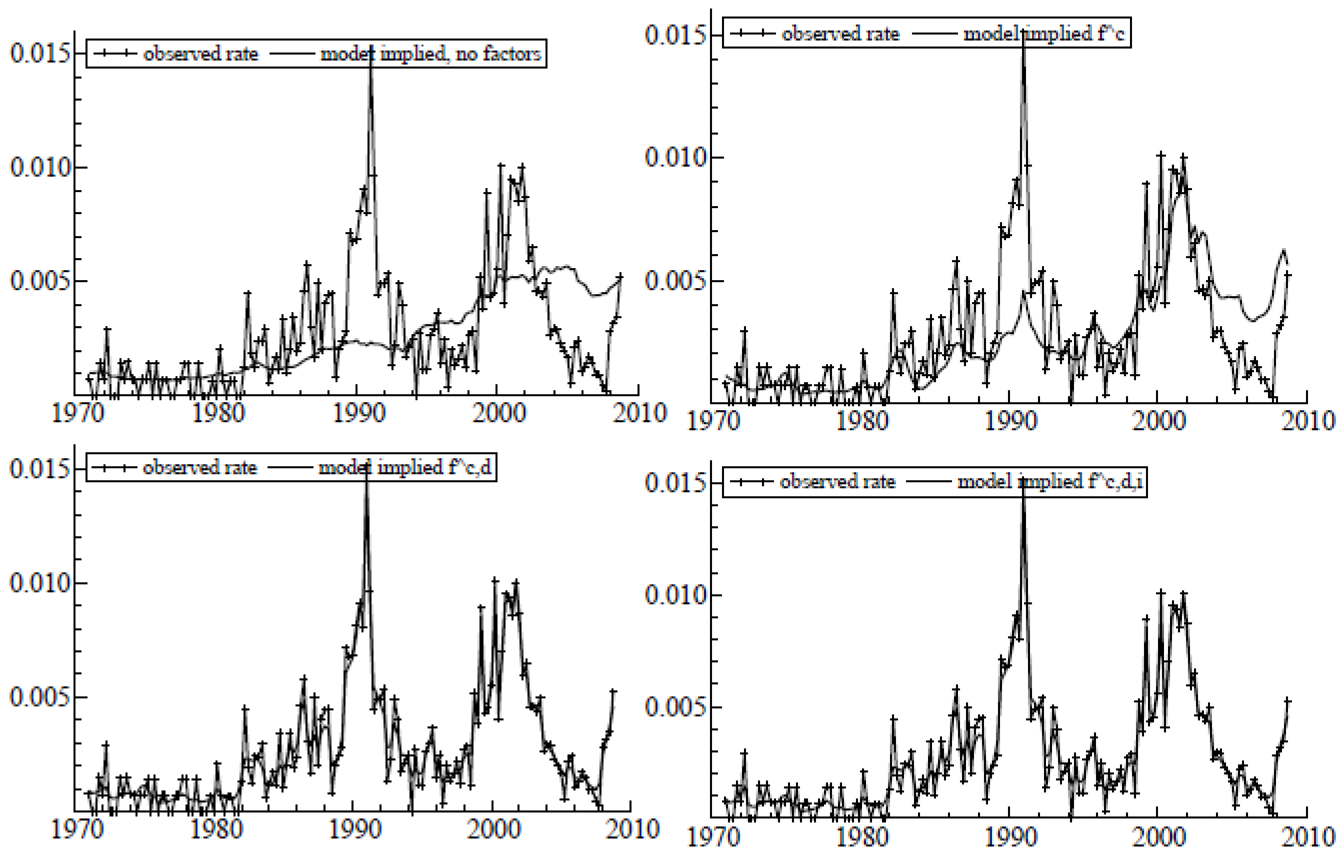
Credit risk (Koopman/Lucas/Schwaab 2009)

Figure 5: Model fit to observed aggregate default rate

Each panel plots the observed quarterly default rate for all rated firms against the default rate implied by different model specifications. The models feature either (a) no factors, (b) only macro factors f^c, (c) macro factors and a frailty component f^c, f^d, and (d) all factors f^c, f^d, f^i, respectively.



Big questions …

- How large is systematic credit risk?
- How much of systematic credit risk can be explained by observable variables?
- What is the composition of the remainder of systematic credit risk? Macro, frailty, contagion, industry, firm-specific
- What are the consequences for portfolio loss distributions and economic capital?

Finale …

- Develop an integrated framework to account for all these features
- Propose a methodology for decomposition and quantified attribution
- Illustrate the deployment of the methodology for estimation, forecasting, and simulation (incl stress testing)
- Include many predictor variables [compared to the literature, that typically uses a small number]

What do we know?

- Defaults (and ratings) covary with the business cycle
- However, the doubly stochastic intensity model assumption is typically rejected (Duffie, Das, Kapadia)
- Additional dynamics in defaults and ratings appear relevant, above and beyond macro, banking, and markets conditions

Recent literature

- McNeil and Wendin (JEF, 2008)
- Koopman, Lucas (2008), Koopman, Lucas, Monteiro (2008), Koopman, Kraeussl, Lucas, Monteiro (2008), Koopman, Lucas, Schwaab (2008, 2009)
- Duffie, Eckner, Horell, Saita (2008)
- Castro (2009)
- Creal, Koopman, Lucas (2009)

Summary of main results

- 3 sources of systematic credit risk important: macro, frailty, contagion/industry
- 30% systematic credit risk: 50% macro, 25% frailty, 25% industry
- Frailty particularly important around stressed periods, and for subinvestment grade
- Macro captures more of investment grade risk
- Omitting frailty may under- or over-estimate risk
- Frailty captures more than missed nonlinear response to business cycle dynamics
- Economic contribution of frailty and industry factors are significant

Set-up

Model

Estimation and inference procedures

Attribution methodology

Forecasting and simulation methodology

Empirical results

Model set-up: state space framework

$$\text{Mixed obs } Y_t = (y_{1t}, \ldots, y_{Jt}, x_{1t}, \ldots, x_{Nt})'.$$

$$\begin{aligned} y_{jt} | f_t^c, f_t^d, f_t^i &\sim \text{Binomial}(k_{jt}, \pi_{jt}) \\ x_{it} | f_t^c &\sim \text{Gaussian}(\theta_{it}, \sigma_i^2) \end{aligned}$$

$$\pi_{jt} = [1 + e^{-\theta_{jt}}]^{-1} \quad \text{Default probability firm j}$$

$$\begin{aligned} \text{Signals } \theta_{jt} &= \lambda_{0,y_j} + \beta_j' f_t^c + \gamma_j' f_t^d + \delta_j' f_t^i \\ \theta_{it} &= \lambda_{0,x_i} + \beta_i' f_t^c \end{aligned}$$

$$\begin{aligned} \text{Factors } f_t &= (f_t^{c\prime}, f_t^{d\prime}, f_t^{i\prime})' \\ &= \Phi f_{t-1} + \eta_t, \quad \eta_t \sim \text{NID}(0, I - \Phi\Phi') \end{aligned}$$

Advantages of the state-space-formulation

- Internally consistent, integrated for in and out of sample
- Embeds well-known models: ARIMA, (dynamic) factor models
- Deals easily with missing data, series of different lengths and / or different frequencies
- Can handle Gaussian and non-Gaussian, linear and non-linear formulations simultatneously
- Successfully deployed in many other areas

Estimation

Problems in a simple PD model

$$y_{jt} \sim \text{binomial}\left(k_{jt}, PD_{jt} = \pi_{jt}\right)$$

$$\theta_{jt} = \left(1 + \exp\left(-\lambda_0 + \beta \cdot f_t\right)\right)^{-1}$$

$$f_{t+1} = Tf_t + \eta_t$$

Updating step cumbersome,

log - likelihood not analytically known

$$\ell = \ln \int \prod_{t=1}^{T} \binom{k_{jt}}{y_{jt}} (\pi_{jt})^{y_{jt}} (1 - \pi_{jt})^{n_{jt} - y_{jt}} \; \mathrm{d}P\left(f_1, \ldots, f_T\right)$$

Simulated Maximum likelihood

$$\ell = \ln \int \prod_{t=1}^{T} \binom{k_{jt}}{y_{jt}} (\pi_{jt})^{y_{jt}} (1-\pi_{jt})^{n_{jt}-y_{jt}} \; \mathrm{d}P\left(f_1, \ldots, f_T\right)$$

$$\hat{\ell} = \ln \frac{1}{S} \sum_{s=1}^{S} \prod_{t=1}^{T} \binom{k_{jt}}{y_{jt}} (\pi_{jt}^{s})^{y_{jt}} (1-\pi_{jt}^{s})^{n_{jt}-y_{jt}}$$

Importance sampling

$$\ell = \ln \int \prod_{t=1}^{T} \binom{n_{jt}}{y_{jt}} (\pi_{jt})^{y_{jt}} (1-\pi_{jt})^{n_{jt}-y_{jt}} \; \mathrm{d}P(f_1, \ldots, f_T)$$

$$= \ln \int \prod_{t=1}^{T} \binom{n_{jt}}{y_{jt}} (\pi_{jt})^{y_{jt}} (1-\pi_{jt})^{n_{jt}-y_{jt}} \; \frac{\mathrm{d}P(f_1, \ldots, f_T)}{\mathrm{d}Q(f_1, \ldots, f_T)} \, \mathrm{d}\, Q(f_1, \ldots, f_T)$$

Intuitive: $q(f) = p(f \mid y)$

Important (!!): Common Random Numbers (CRNs)

Approach: choose sampler to minimize the variance of the

log-likelihood ratio $\dfrac{\mathrm{d}P(f_1, \ldots, f_T)}{\mathrm{d}Q(f_1, \ldots, f_T)}$

Importance sampling

$$\ell = \ln \int \prod_{t=1}^{T} \ell(y \mid f) \, \frac{\mathrm{d}P\left(f_1, \ldots, f_T\right)}{\mathrm{d}Q\left(f_1, \ldots, f_T\right)} \, \mathrm{d}\, Q\left(f_1, \ldots, f_T\right)$$

$$= \ln \int \ell(y \mid f) \cdot w(f) \, \mathrm{d}\, Q\left(f_1, \ldots, f_T\right)$$

$$\approx \frac{1}{S} \sum_{s=1}^{S} \ell(y \mid f^{s}) \cdot w(f^{s})$$

$$\ell(y \mid f) = \prod_{t=1}^{T} \binom{n_{jt}}{y_{jt}} (\pi_{jt})^{y_{jt}} (1 - \pi_{jt})^{n_{jt} - y_{jt}} \quad ,$$

$$w(f) = \prod_{t=1}^{T} \frac{\mathrm{d}P\left(f_1, \ldots, f_T\right)}{\mathrm{d}Q\left(f_1, \ldots, f_T\right)}$$

Example: importance sampling at work

$$y_{it} = f_t + 3 \cdot \varepsilon_{it}, \quad i = 1,...,5, \quad t = 1,...,2000$$

$$f_t = 0.8 f_{t-1} + \eta_t$$

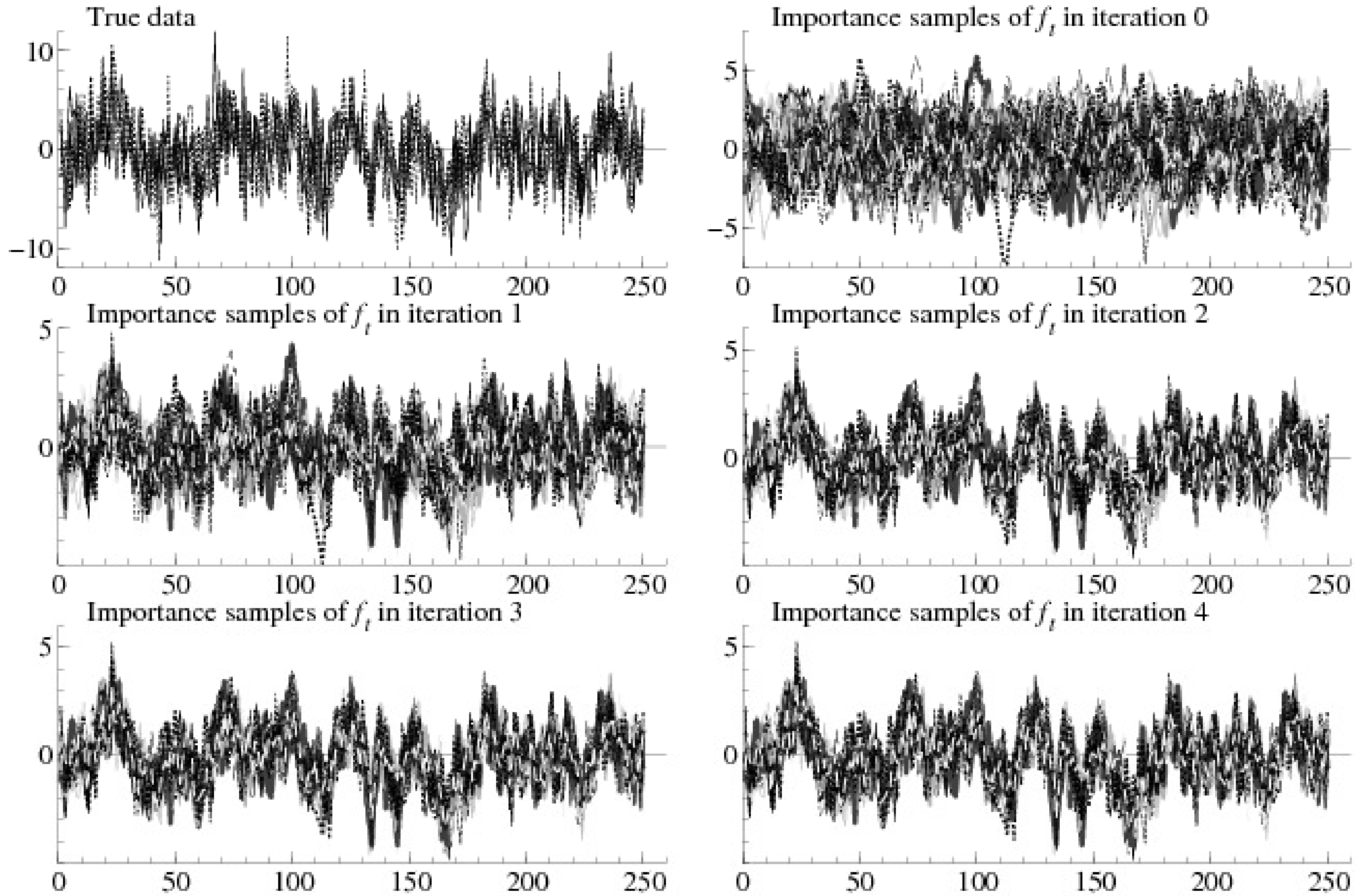
$S = 50$ [EIS importance samples]

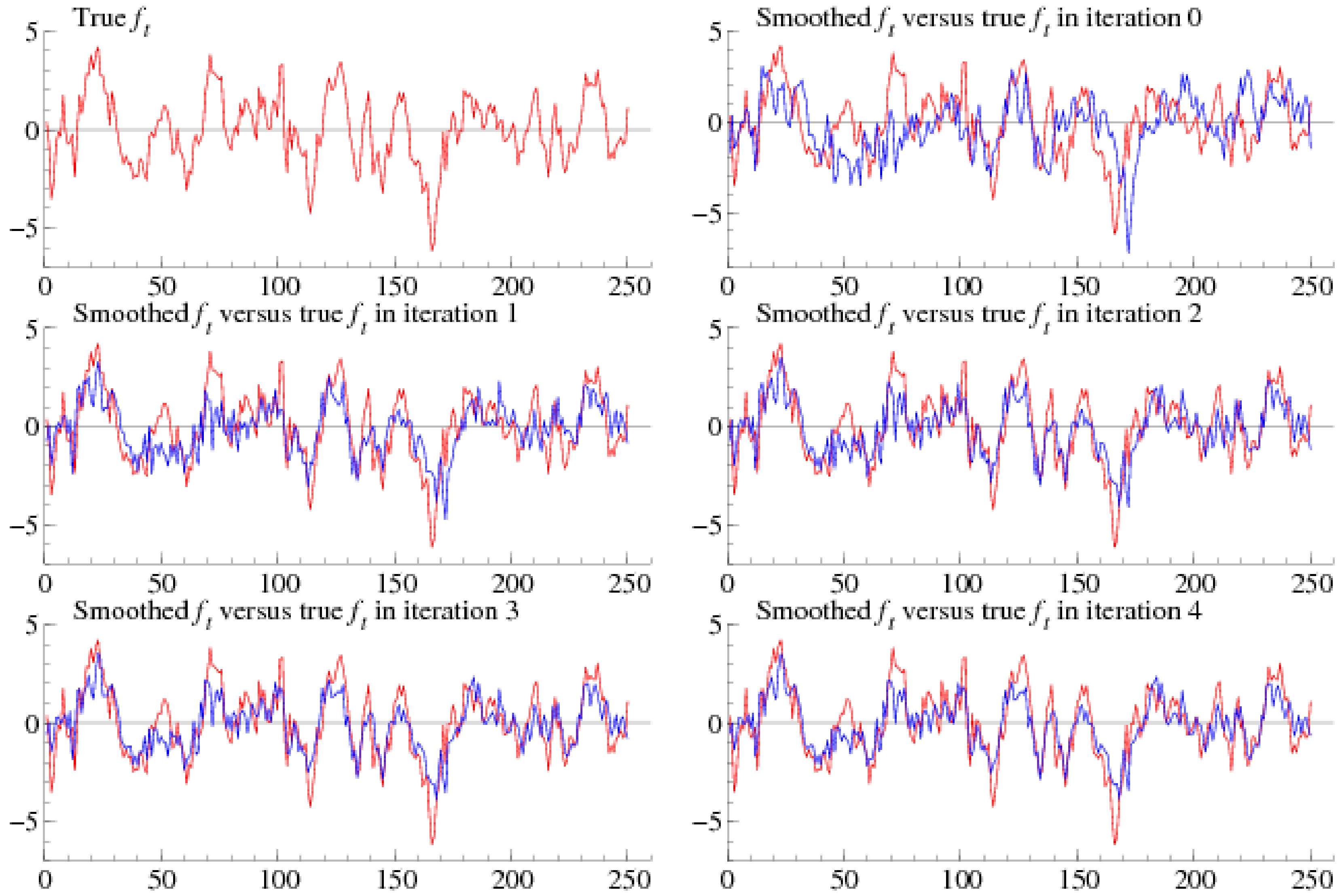
Importance sampler is Gaussian, optimal approximation determined via iteration scheme.

Reference:

Richard and Zhang (J.Econometrics)

Durbin and Koopman (OUP)


True data
Importance samples of f_t in iteration 0
Importance samples of f_t in iteration 1
Importance samples of f_t in iteration 2
Importance samples of f_t in iteration 3
Importance samples of f_t in iteration 4



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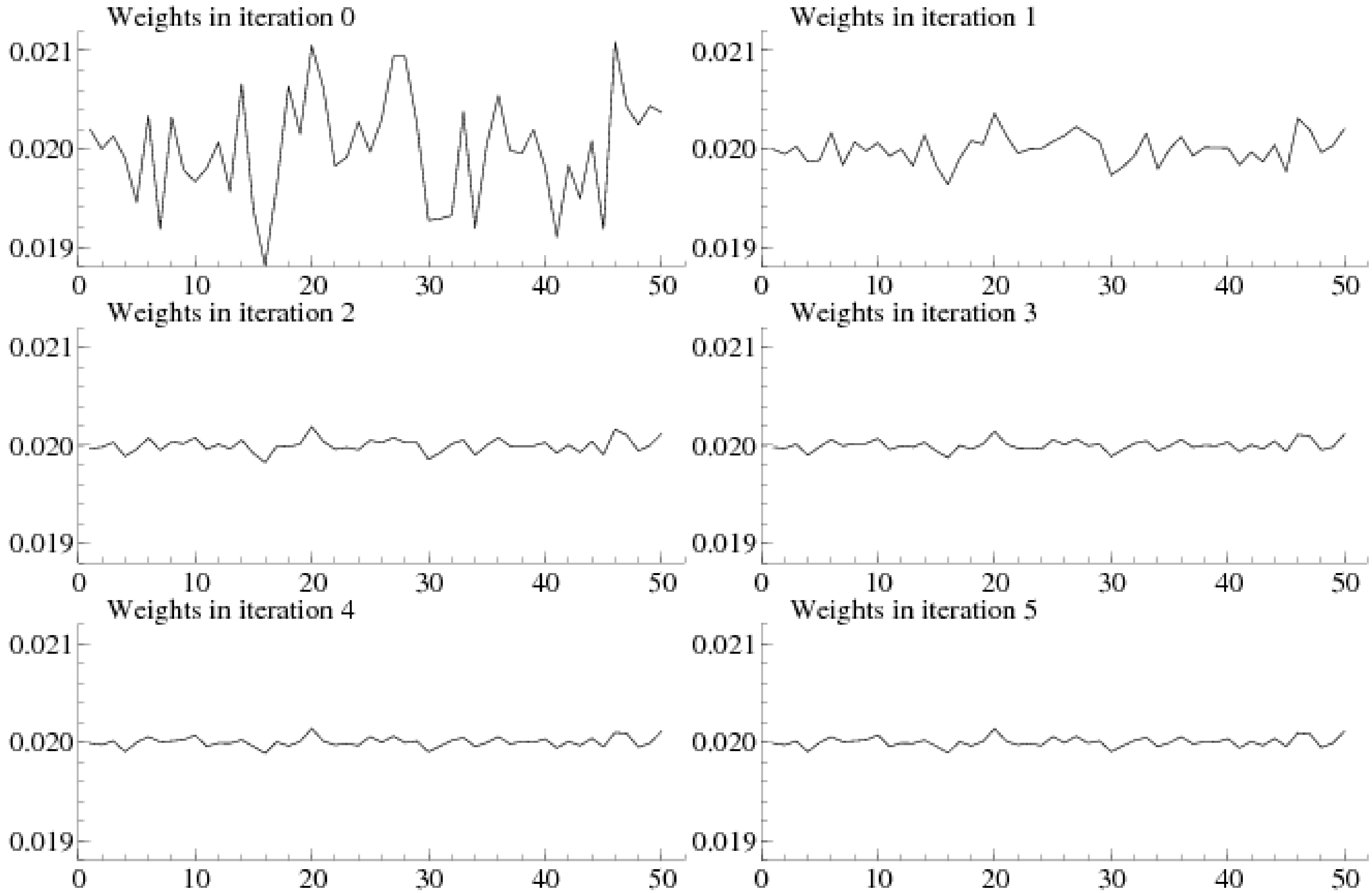

Weights in iteration 0
Weights in iteration 1
Weights in iteration 2
Weights in iteration 3
Weights in iteration 4
Weights in iteration 5
0.021
0.020
0.019
0
10
20
30
40
50

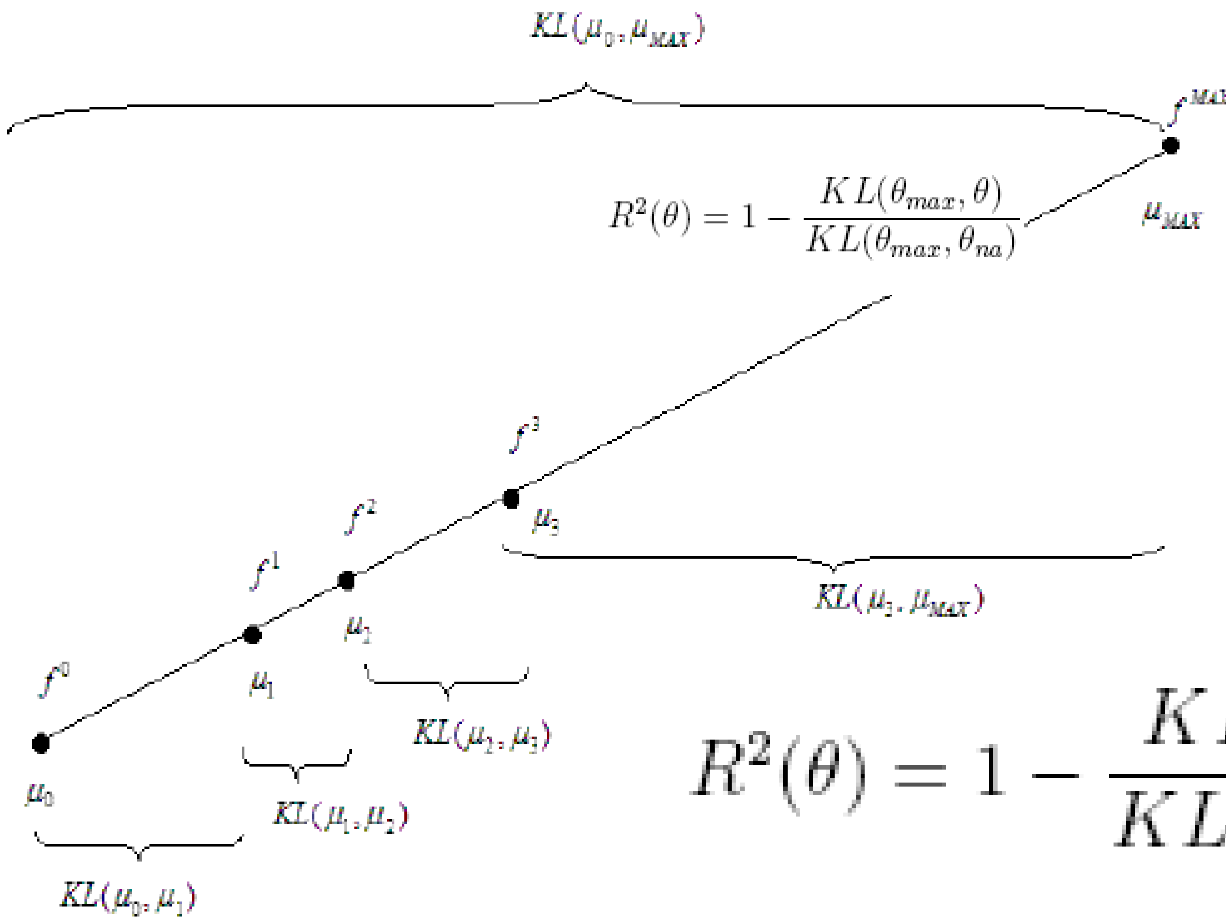
Attribution methodology

Problem

- No R^2 measure readily available
- Data is mixed measurement, mixed frequency
- Cameron & Windmeijer: likelihood based fit criteria
- Kullback-Leibler distance between distributions
- For linear Gaussian model, the standard R^2 is recovered, for binary data the McFadden pseudo-R^2 is recovered

Attribution methodology

$$\mathrm{KL}(\mu_1, \mu_2) = 2 \int \log \left(f_{\mu_1}(y) / f_{\mu_2}(y) \right) f_{\mu_1}(y) dy$$



$$R^2(\theta) = 1 - \frac{KL(\theta_{max}, \theta)}{KL(\theta_{max}, \theta_{na})}.$$

Forecasting and simulation

No auxiliary models needed

$$\text{Mixed obs } Y_t = (y_{1t}, ..., y_{Jt}, x_{1t}, ..., x_{Nt})'.$$

$$\begin{aligned} y_{jt} | f_t^c, f_t^d, f_t^i &\sim \text{Binomial}(k_{jt}, \pi_{jt}) \\ x_{it} | f_t^c &\sim \text{Gaussian}(\theta_{it}, \sigma_i^2) \end{aligned}$$

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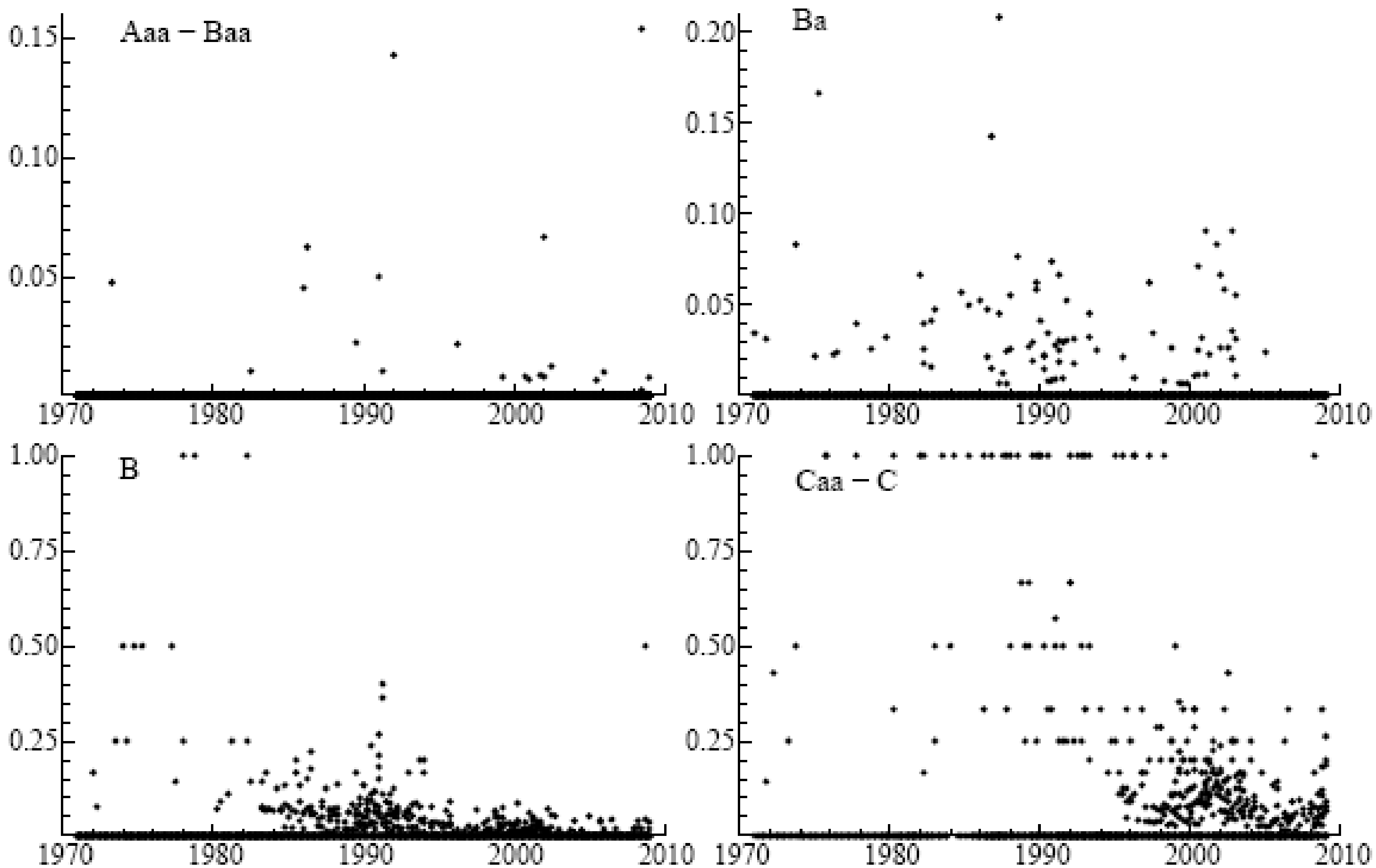
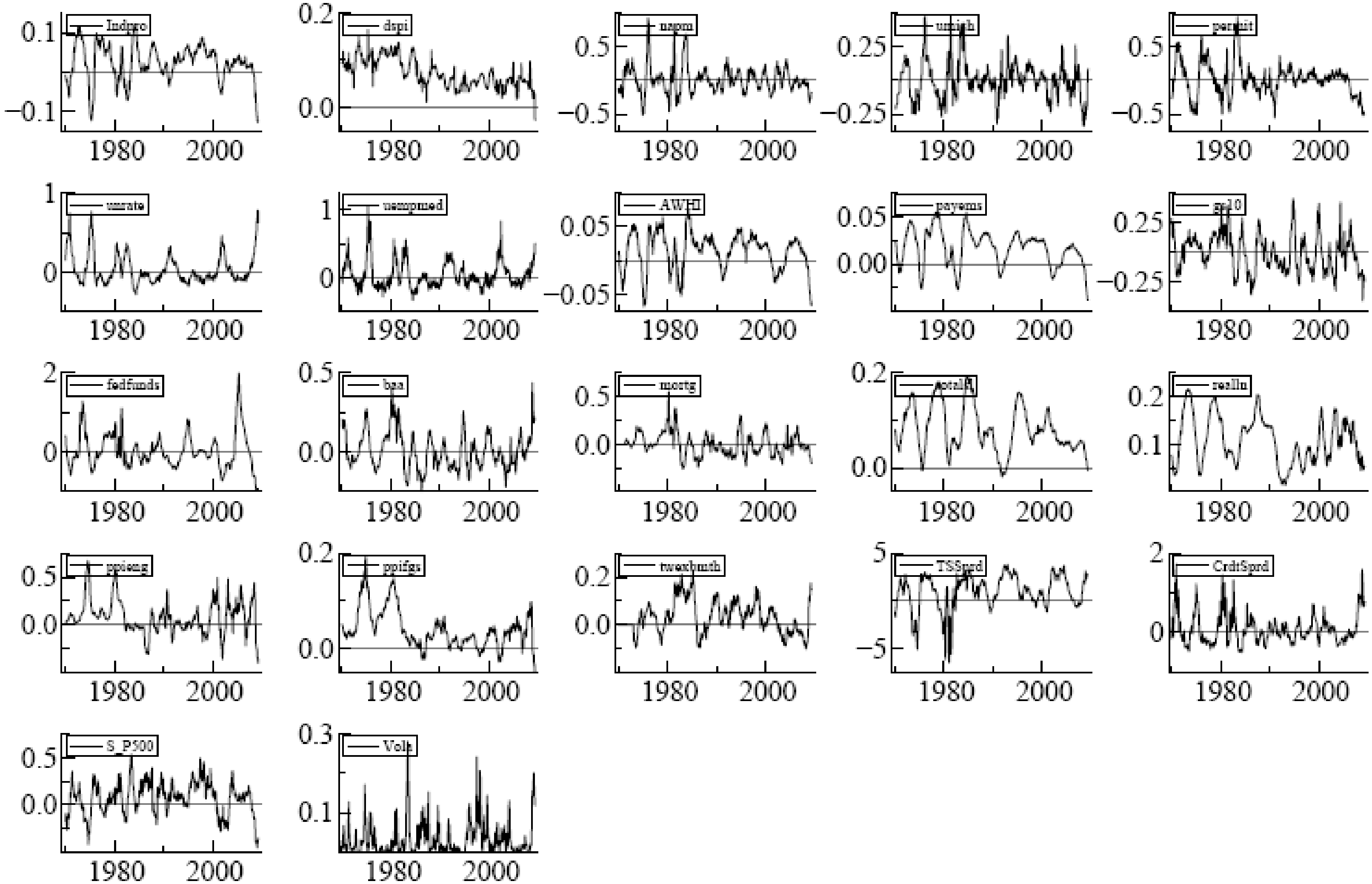
Empirical results

The default data



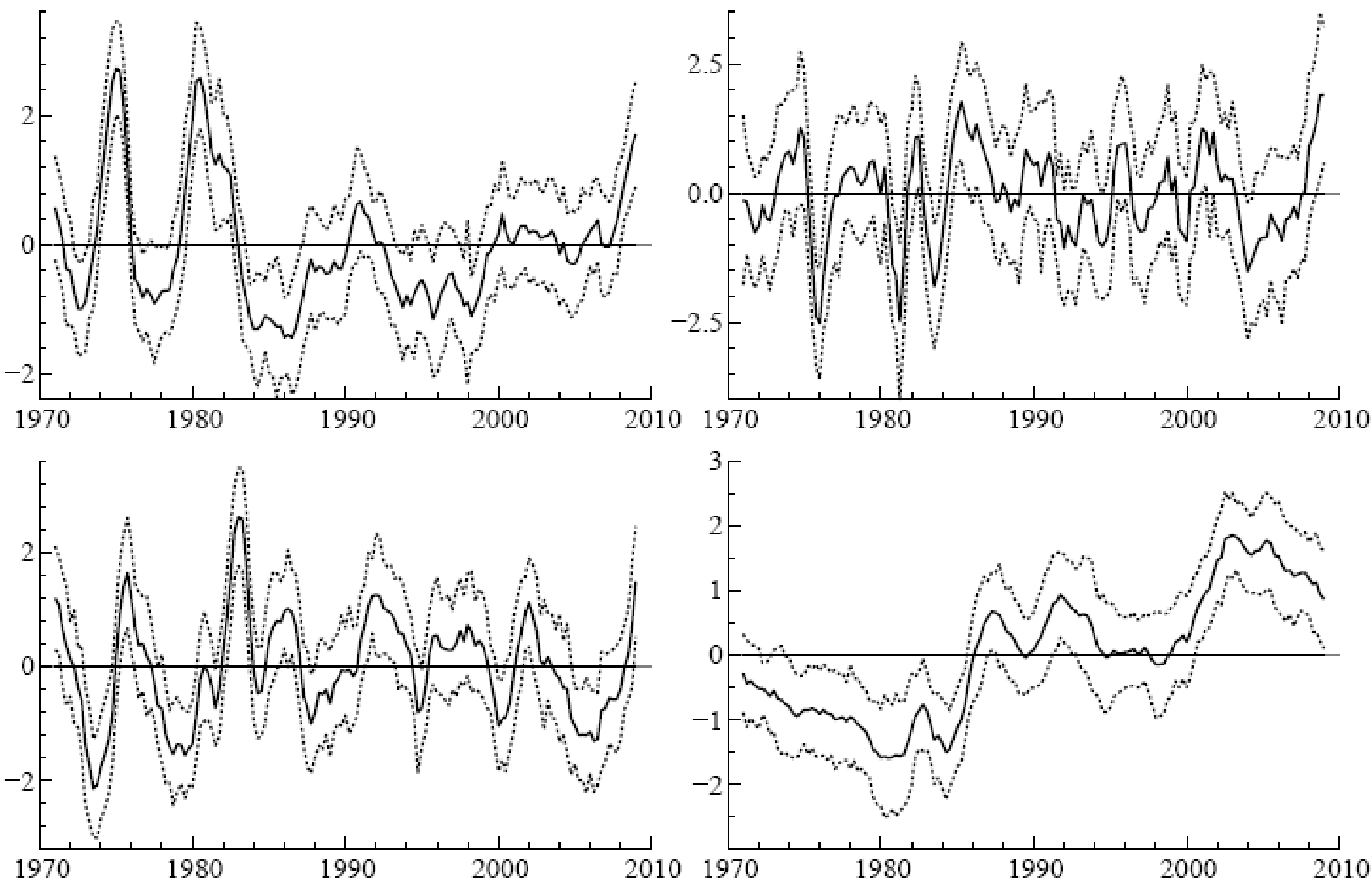
The macro data



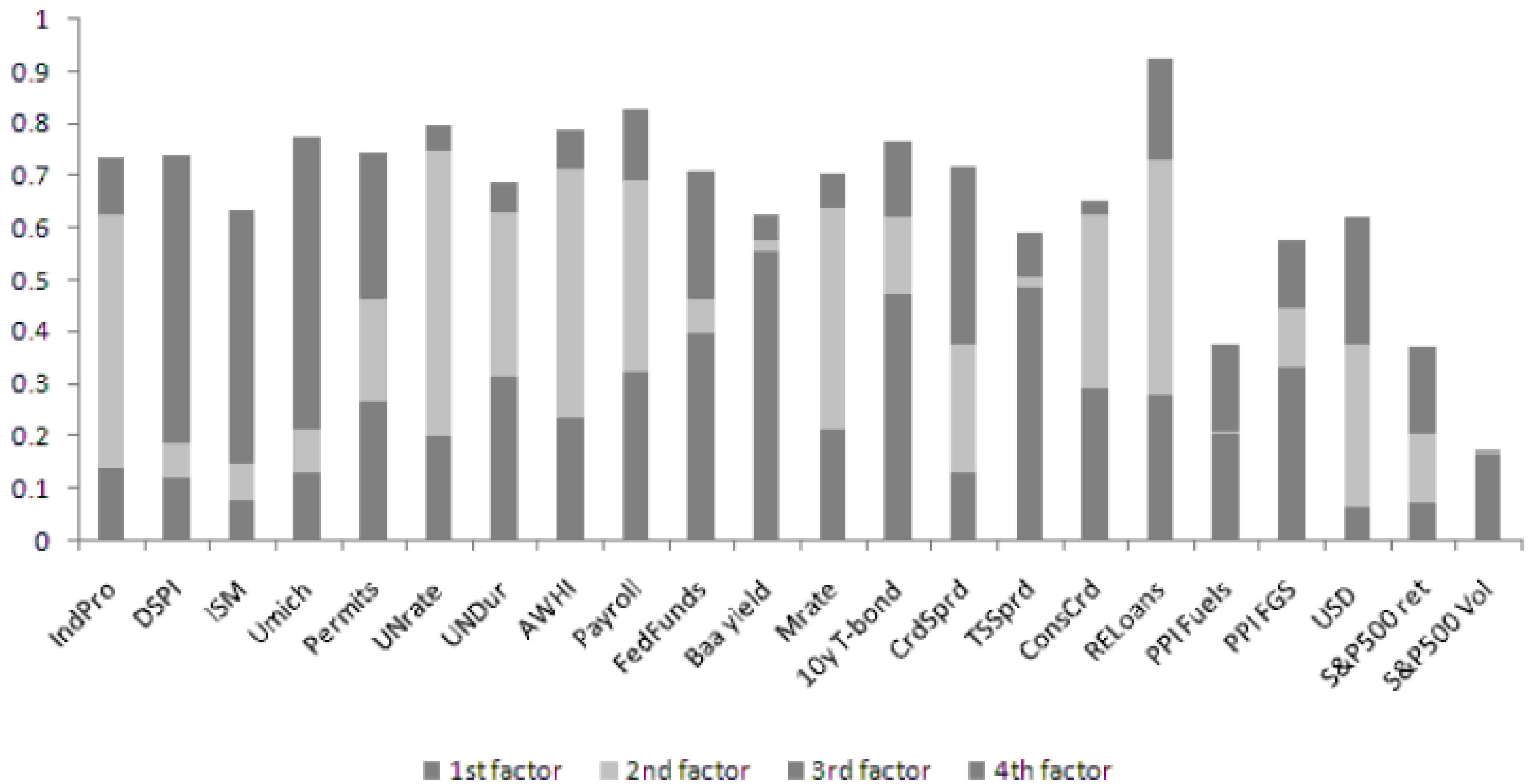
Estimation results

Intercepts λ_j		Loadings f_t^m		Loadings f_t^d		Loadings f_t^i	
par	val	par	val	par	val	par	val
λ_0	-2.52	$\beta_{1,IG}$	0.30	γ_{IG}	0.20	δ_{fin}	0.64
		$\beta_{1,Ba}$	0.20	γ_{Ba}	0.51	δ_{tra}	0.74
$\lambda_{1,fin}$	-0.24	$\beta_{1,B}$	0.29	γ_B	0.64	δ_{lei}	0.41
$\lambda_{1,tra}$	-0.16	$\beta_{1,C}$	0.19	γ_C	0.42	δ_{utl}	0.99
$\lambda_{1,lei}$	-0.17					δ_{tec}	0.39
$\lambda_{1,utl}$	-0.51	$\beta_{2,IG}$	0.24			δ_{ret}	0.43
$\lambda_{1,tec}$	-0.96	$\beta_{2,Ba}$	0.18				
$\lambda_{1,ret}$	-0.33	$\beta_{2,B}$	0.10				
		$\beta_{2,C}$	0.26				
$\lambda_{2,IG}$	-7.13						
$\lambda_{2,BB}$	-3.89	$\beta_{3,IG}$	0.66				
$\lambda_{2,B}$	-2.12	$\beta_{3,Ba}$	0.40				
		$\beta_{3,B}$	0.27				
		$\beta_{3,C}$	0.19				
		$\beta_{4,IG}$	0.66				
		$\beta_{4,Ba}$	0.26				
		$\beta_{4,B}$	-0.26				
		$\beta_{4,C}$	-0.02				

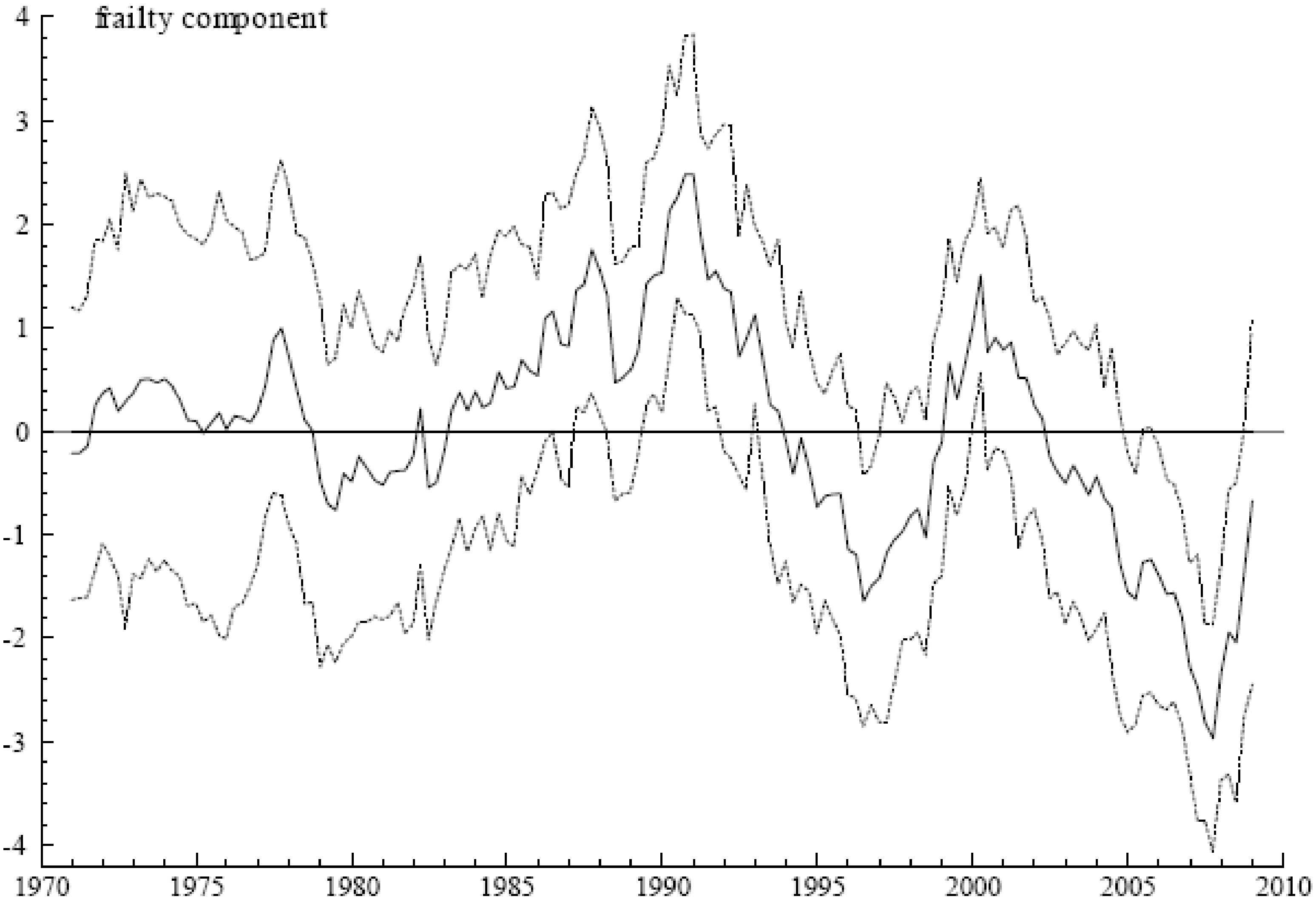
Smoothed macro factors



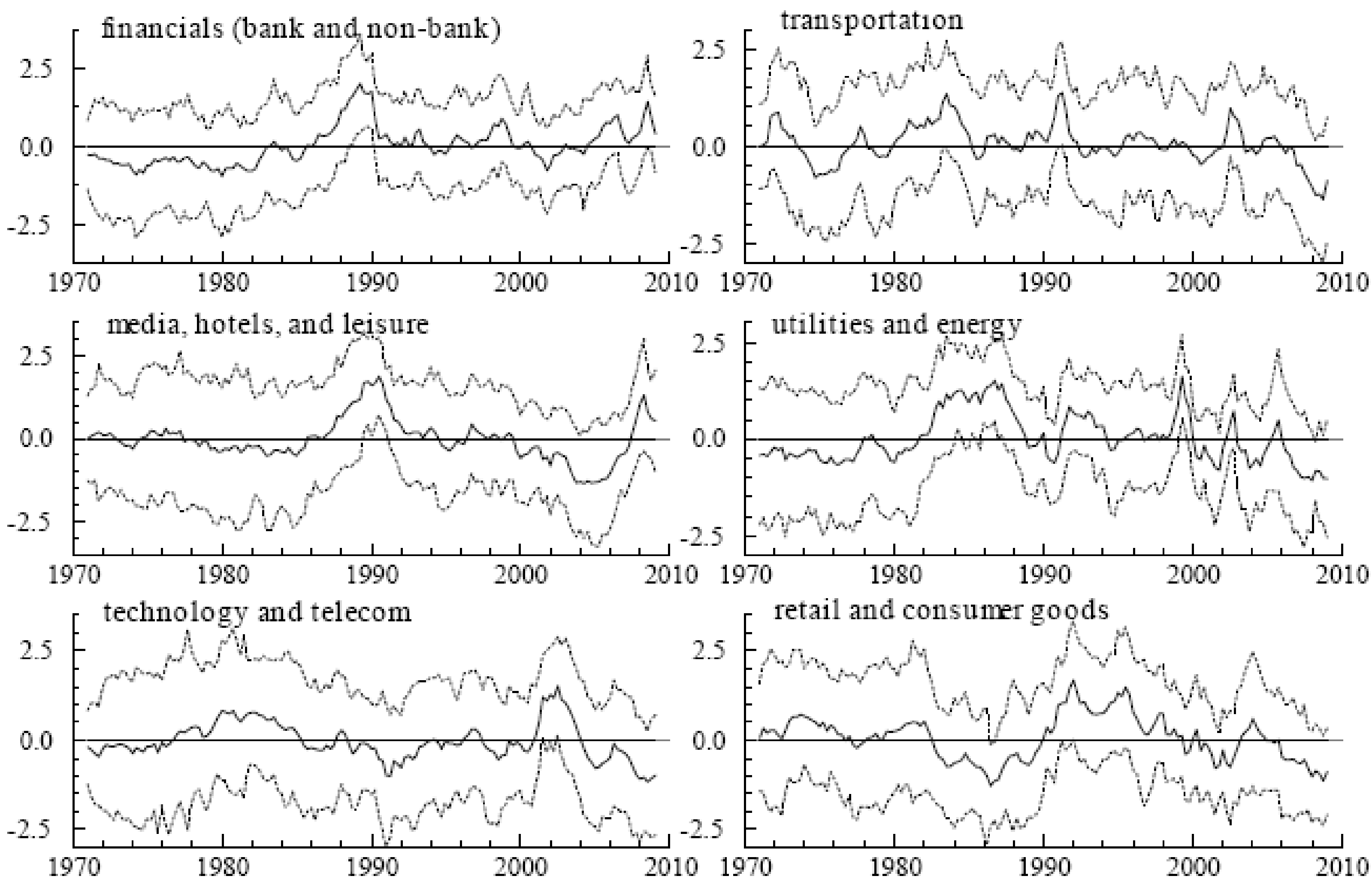
Shares of explained variation by the 4 factors



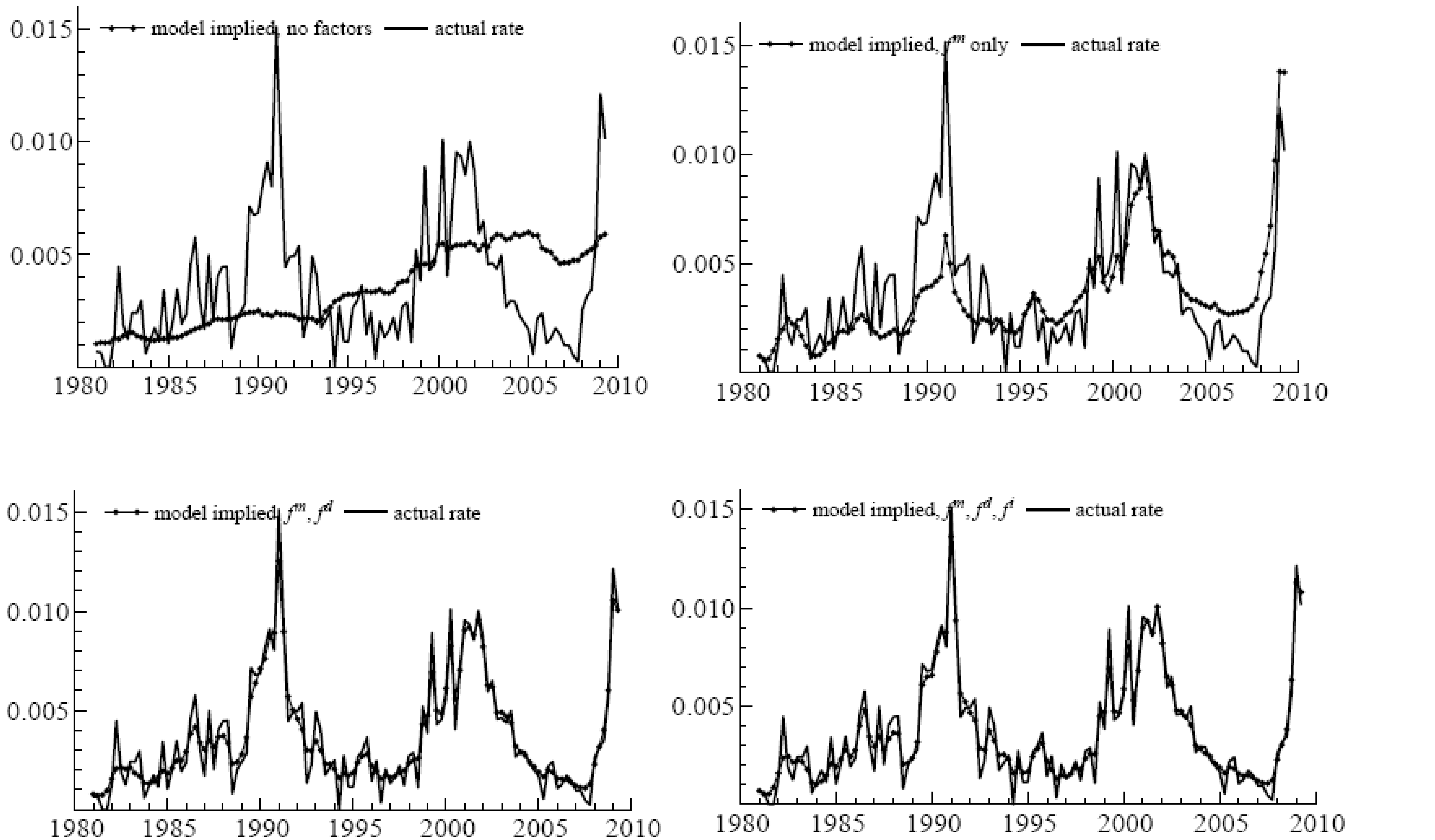
Estimated frailty component



Estimated industry factors



Model fit


model implied, no factors
actual rate
model implied, f^m only
actual rate
model implied, f^m, f^d
actual rate
model implied, f^m, f^d, f^i
actual rate
0.015
0.010
0.005
1980
1985
1990
1995
2000
2005
2010

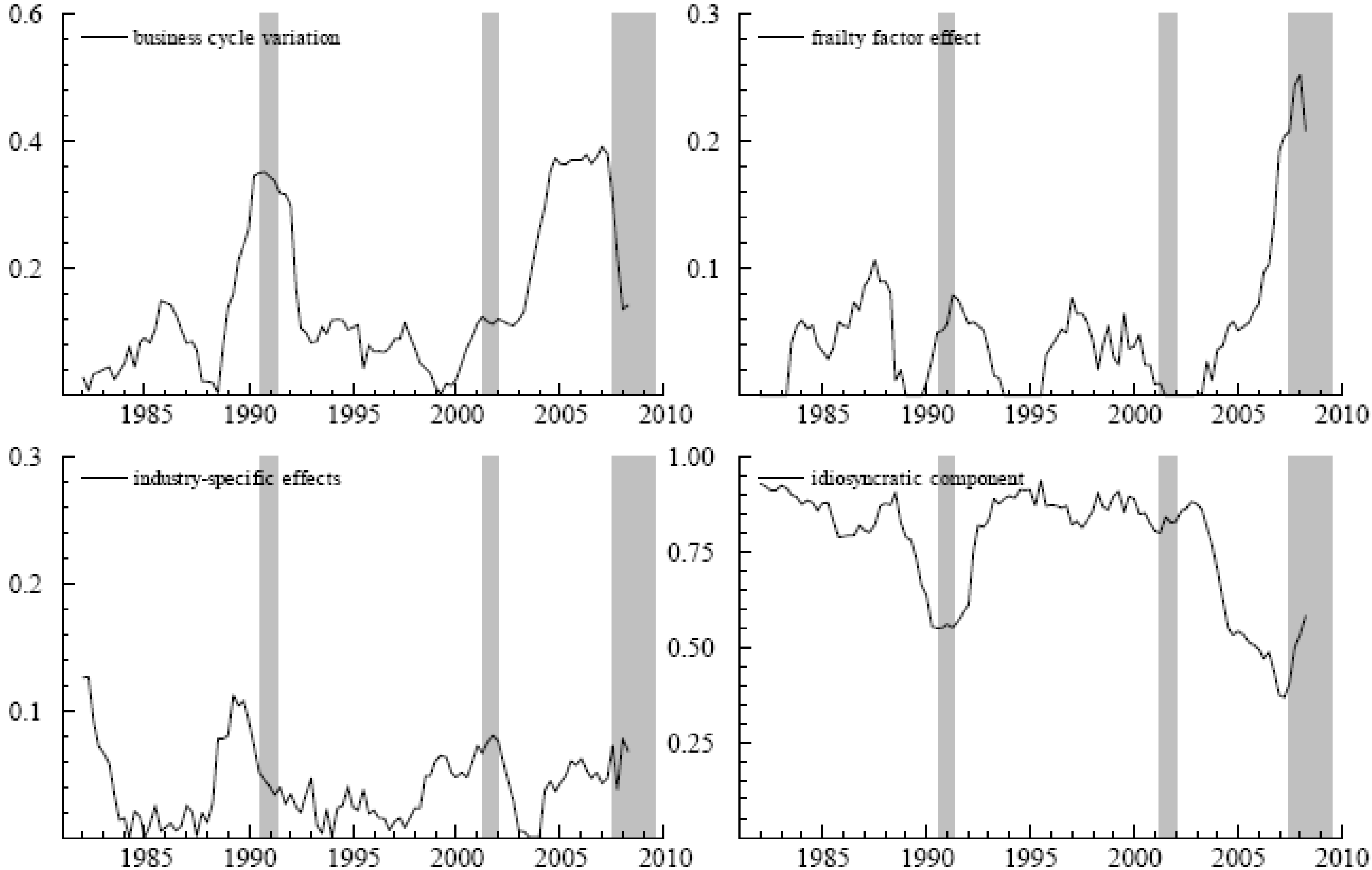
A decomposition of total risk

Data	Business cycle f_t^c	Frailty risk f_t^d	Industry-level f_t^i	Idiosyncratic *distr.*
Pooled	11.4%	13.9%	8.6%	66.1%
	(33.6%)	(40.9%)	(25.4%)	
Rating groups:				
Aaa-Baa	10.4%	1.1%	6.4%	82.1%
	(58.0%)	(6.3%)	(35.7%)	
Ba	7.1%	7.5%	6.2%	79.2%
	(34.0%)	(36.0%)	(30.0%)	
B	12.5%	22.3%	7.0%	58.2%
	(30.0%)	(53.2%)	(16.8%)	
Caa-C	12.3%	8.9%	12.3%	66.5%
	(36.7%)	(26.5%)	(36.8%)	

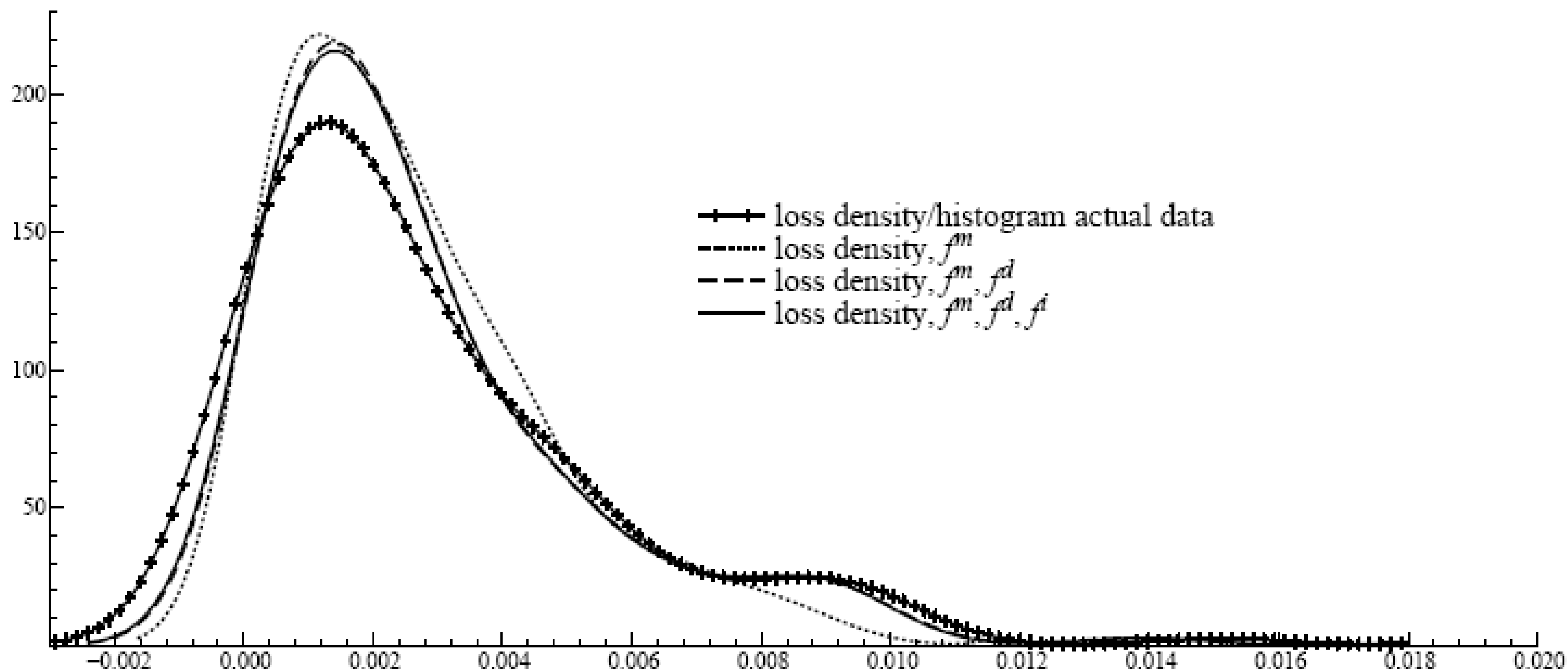
A decomposition of total risk

Data	Business cycle f_t^c	Frailty risk f_t^d	Industry-level f_t^i	Idiosyncratic *distr.*
Industry sectors:				
Bank	5.4%	11.9%	18.8%	63.8%
Financial non-Bank	5.0%	5.3%	9.2%	80.5%
Transportation	7.4%	13.7%	18.8%	60.1%
Media	10.6%	19.9%	8.8%	60.8%
Leisure	15.7%	11.1%	2.6%	70.7%
Utilities	1.1%	4.9%	10.7%	83.3%
Energy	24.0%	8.7%	18.0%	49.3%
Industrial	16.3%	23.1%	-	60.7%
High Tech	17.2%	11.0%	12.5%	59.3%
Retail	6.7%	9.6%	10.4%	73.2%
Consumer Goods	4.6%	18.4%	1.3%	75.7%
Misc	4.5%	13.2%	1.4%	80.9%

Time varying risk shares (8m rolling)



Model implied loss distributions (in sample fits)



Predictive loss distribution

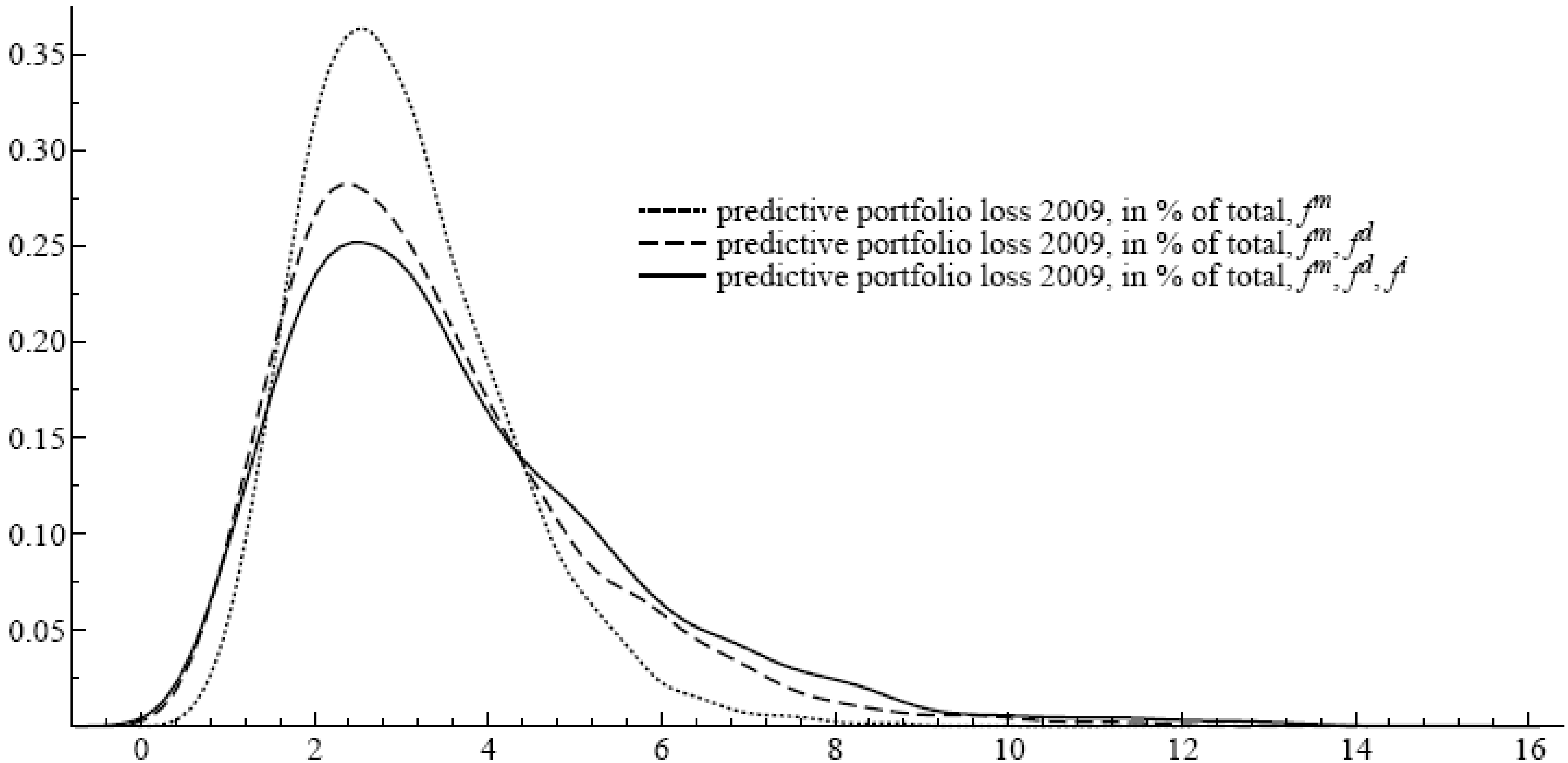
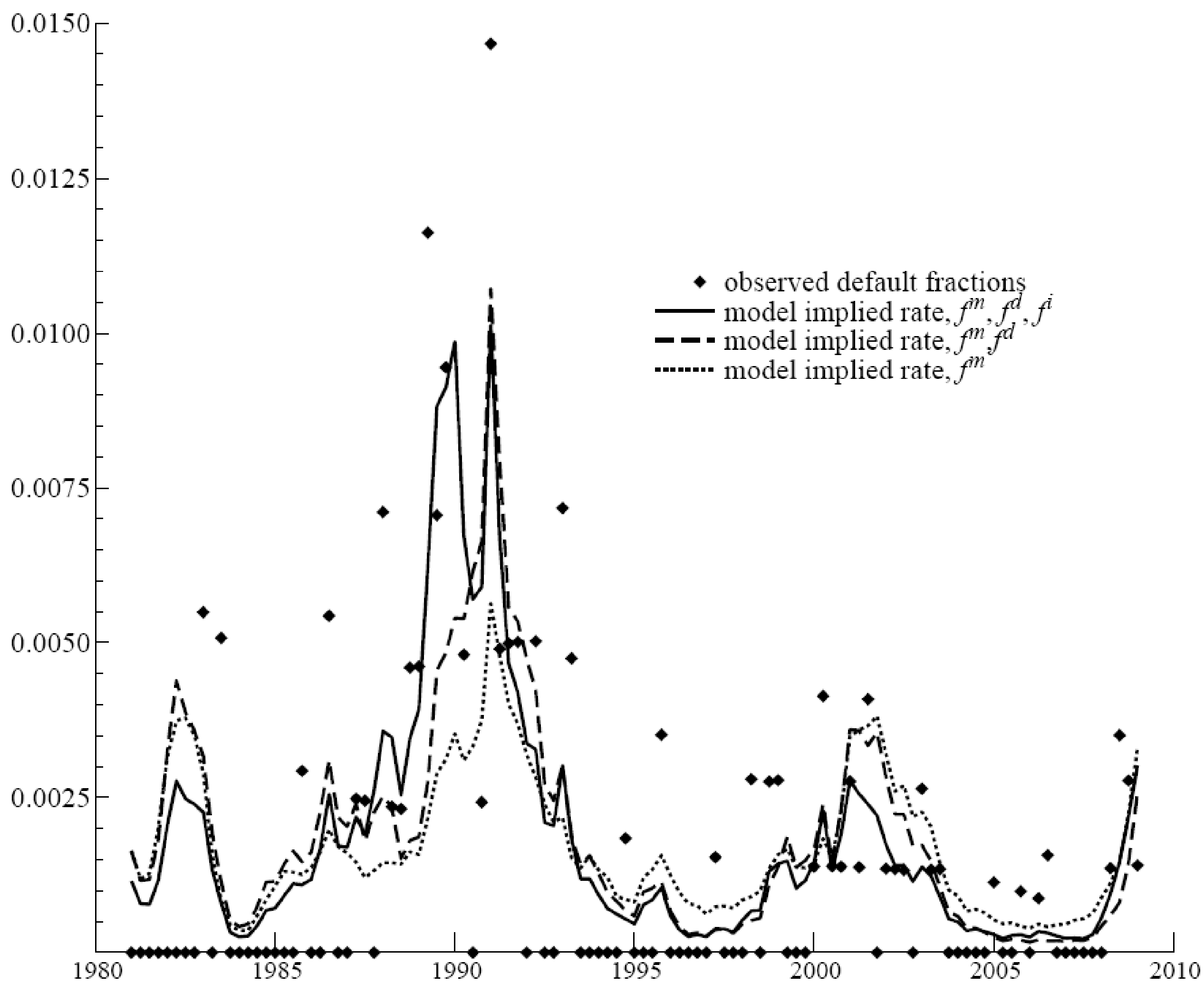


Figure 10: Quarterly time-varying default intensities for financial firms

We plot smoothed estimates of quarterly time-varying default rate for the financial sector. We distinguish a model with (i) common variation with macro data only, (ii) macro factors and a frailty component, and (iii) macro factors, frailty component, and industry-specific factors, respectively. The model-implied quarterly rates are graphed against the observed default fractions for financial firms.



Conclusions

- Frailty models are a new and flexible class of models to
  - Account for default characteristics
  - Account for clustering / concentration risk
  - Account for dependence on macro
  - Offer an integrated framework for fitting, forecasting, and simulation
- Implementation drawbacks
  - Parameter estimation needs time series dimension (is it there?)
- Extensions: easier estimation (GAS), PD&LGD integration