Eurobanking 2006, May 21st-24th, Dubrovnik

Interest rate derivative pricing from an auditors point of view

IKB Deutsche Industriebank

Disclaimer

The contents of this presentation are the sole and personal opinion of the author and do not express IKBs opinion on any subject presented in the following.

Contents

Where I work

Group Audit

- Team 1 — Business processes Corporate Lending, Real Estate Finance, Private Equity
- Team 2 — Business processes Structured Finance, Treasury, Leasing
- Team 3 — Mathematics & IT
  - Credit pricing
  - Credit portfolio models
  - Derivatives pricing
  - Market risk models
  - …

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Task

When auditing market risk management the question arises if the prices calculated for the
- market conformity check and for the
- daily assessment of market risk (value at risk) are reasonable w.r.t. a defined tolerance.

A good test for this are the matchings of deal prices done in the cash collateral management with counterparting banks by group operations.

It is important, however, for an auditor to have his own opinion what reasonable prices are. In addition group audit should be able to calculate these prices completely independent of the banks pricing systems in treasury and risk management.

Task

More specifically the task was defined as follows:

- Check out what models are available for pricing structured interest rate swaps
- Set up some models and price some types of traded swaps only by using elementary tools
- Check if risk management's prices are in line with self calculated prices
- Try to find the crucial points in the pricing procedure which can go wrong
- Try to gain an understanding which are the right models for which types of swaps and why (model risk!)

Contents

General pricing approach

The general pricing formula in complete and arbitrage free markets is

Price(today) / Numeraire (today) = E (Price (T) / Numeraire (T))

The numeraire is used to discount the asset prices and can be chosen freely (but >0). It defines a probability measure w.r.t. which all discounted asset prices are martingales, i.e. the discounted price today is equal to the future expectation. This measure is called the risk neutral measure. Thus, the price calculated by the formula above is the price of setting up a perfect hedging portfolio today for the instrument in question.

In what follows the numeraire is always chosen as the bank account B(t) with dynamics

$$dB(t) = r(t)\, B(t)\, dt$$

where r(t) is the short rate process which is generally stochastic.

Complexity levels of interest rate swaps

Attempt to roughly classify IRS w.r.t. the complexity of pricing them:

Level 1: „Use forward rates for variable rates and discount on today's yield curve.“
Correct, if payoff is „deterministic nominal x rate“, made standard in advance, no quanto elements are present, rate is not a derived rate (e.g. a swap rate)

Level 2: „As level 1 but make certain convexity adjustments or use Black76-Formula.“
Correct, if payments are made in arrears, quanto effects are present, rate is a derived rate (e.g. a swap rate) or standard european option payoffs („max(Price-Strike,0)“) are present.

Level 3: All what is beyond level 2.
e.g. nominal is accreting or amortizing dependent on future rates, bermudan or american optional elements are included, payoff is dependent on quantities other than in level 1 or 2, e.g. on the volatility of certain rates or a spread between rates with different maturities.

Interest rate models

Level 1 and 2 are supposed to be standard, easy to calculate and not problematic, thus we will not consider swaps at these levels further (although there are still non-trivial aspects e.g. the construction of the yield curve at level 1 or the correlation estimates for convexity adjustments at level 2).

For the pricing at Level 3 models for the evolution of the yield curve are needed.

These models can be classified roughly as
- short rate models and
- forward rate models.

Forward rate models / The HJM Framework

Forward rate models describe the dynamics of instantaneous forward rates f(t,T).
Heath, Jarrow and Morton showed in 1992 that (continuous) arbitrage free models must satisfy

$$df(t,T) = \sigma(t,T)^T \left(\int_t^T \sigma(t,s)ds\right)dt + \sigma(t,T)^T dW(t)$$

where $\sigma(t,T)$ is a family of vector processes describing the volatility structure and W is a vector brownian motion.

When computing this dynamic, however, the state variable to simulate is the whole forward curve $T \to f(t,T)$, which results in a high computational effort.

Short rate models have a much easier structure as we will see later on.

Forward rate models / Market models

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Brace, Gatarek and Musiella modified this approach by using market rates such as 3-month Libor forward rates instead of instanteous forward rates (Libor Market Model, LMM). The dynamic can be stated as follows:

$$dF_k(t) = \sigma_k(t) F_k(t) dZ_k(t)$$

for $t < T_{k-1}$ where $F_k(t)$ is the forward rate at time t for the period $[T_{k-1}; T_k]$, $\sigma_k(t)$ is the volatility of F_k and $Z_k(t)$ is the k-th component of a M-dimensional brownian motion with instanteous correlation structure given by an MxM matrix ρ.

This dynamic is expressed w.r.t. the T_k-Forward Measure (i.e. the numeraire is $P(t,T_k)$).

M is the number of forward rates needed in the model. E.g. when pricing a derivative with 30y to maturity and using 6m forward rates, M=60.

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Forward rate models / Market models

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The LMM model can relatively easily be calibrated to the swaption matrix by an analytical „cascade" technique, where only quadratic equations need to be solved.
To fully specify the model,however, an exogenous correlation matrix is needed which can be introduced by historical estimations combined with certain parametric forms of this matrix[1].

Note 1: It is known, that low-rank matrices (i.e. a low number of factors in the model) lead to
- to high correlations between adjacent forward rates and
- to low correlations between distant forward rates[2]

Note 2: Another version models swap rates (Swap Market Model, SMM).
The LMM is directly compatible with cap / floor market implied volatilities.
The SMM is directly compatible with swaption market implied volatilities.

At this stage of the task, LMM and SMM is not implemented.

[1] See for example Morini, Brigo: „New developments on the analytical cascade swaption calibrations of the LMM"
[2] See Brigo, Mercurio: „Interest Rate models in theory and practice"

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Extra slide

Market models: The Swap Market Model

Another version models forward swap rates (Swap Market Model, SMM) instead of Libor rates.

The market models are called "market models" because they

- directly model rates observable in the interest rate markets
- are directly compatible with the market's pricing formulas for caps and swaptions

i.e. the vola of a Libor- / Swap - Forward Rate is the same as in the
Black 76 - formula used by the market to price caps / swaptions.

BUT: The LMM and the SMM are not compatible !
(i.e. in the LMM swap rates are not lognormal and in the SMM Libor rates are not lognormal.)

Extra slide

Calibration of the LMM

The LMM model can relatively easliy be calibrated to the swaption matrix by an analytical „cascade“ technique, where only quadratic equations need to be solved.

To fully specify the model,however, an exogenous correlation matrix is needed which can be introduced by historical estimations combined with certain parametric forms of this matrix[1].

[1] See for example Morini, Brigo: „New developments on the analytical cascade swaption calibrations of the LMM“

Extra slide

Low factor LMMs

Note: It is known, that low-rank matrices (i.e. a low number of factors in the model) lead to

- to high correlations between adjacent forward rates and
- to low correlations between distant forward rates[1]

This has to be thought of carefully when e.g. pricing spread components such as 30y - 2y swap rates (see above).

[1] See Brigo, Mercurio: „Interest Rate models in theory and practice“

Extra slide

Smiles in market models

The standard LMM / SMM do not produce any smile!

This is due to their construction which follows the Black 76 - formula.

Therefore smiles in market models have to be introduced by extensions of the model, e.g.

- local volatility models (where the vola may depend on the modelled rate)

- stochastic volatility models (where the vola itself is a stochastic process)

Short rate models

Short rate models describe the dynamics of the short rate r(t), thereby determining the dynamics of the whole yield curve over time (because P(t,T) = E (B(t) / B(T))).

This model class includes among others the	Limitations:
- Vasicek model (1977)	Does not fit initial term structure!*
- Cox, Ingerson Ross model (1985)	Does not fit initial term structure!*
- Ho-Lee (1986)	No mean reversion!
- Hull White (1990) & Generalizations	Negative rates possible (but with low prob.)

In the following we will concentrate on the Hull White model.

*as a consequence, even simple Bonds are mispriced in these models, so there is not much confidence, that IRS, whose prices in general are even more sensitive to the underlying rates priced reasonable.

The Hull White Model

The Hull White Model can be stated as the following stochastic differential equation:

$$dx = - a\, x(t)\, dt + \sigma\, dW$$
$$r(t) = x(t) + \phi(t)$$

a	= Mean reversion parameter
σ	= Volatility
W	= Standard brownian motion
ϕ	= Fitting parameter chosen to fit the initial yield curve

A second factor W‘ can easily be added by introducing

$$dy = - b\, x(t)\, dt + \eta\, dW'$$
$$r(t) = x(t) + y(t) + \phi(t)$$

with (constant) instantaneous correlation ρ between W and W‘.

Why more than one factor / how many?

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It can be shown that in the case of one factor rates with different maturities have always a correlation of +1.

This is not necessarily a problem when the payoffs only depend on the marginal distributions of rates with different maturities.

But this will definitely be a problem, if for instance the payoff depends on the spread between short- and long-term rates. We will come to this later.

Furthermore, principal component analysis of the yield curve shows*, that the yield curve is very well explained by 3 factors (95%) and still well explained by 2 factors (90%); One factor explains apprx. 75%.

*See Brigo, Mercurio: „Interest Rate models in theory and practice“

Extra slide

Smiles in the Hull White approach

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The Black 76 - pricing formula assumes lognormal forward libor / swap - rates.
The market does not follow this assumption but in fact assumes different distributions.
In order to compensate this discrepancy, smiles occur in the market quotes of implied volatilities, i.e. the volas are made dependent of the cap's / swaption's strike price.

We have only used "at the money" instruments to calibrate our models.

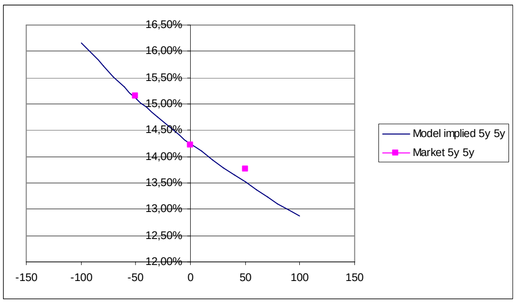
Because the Hull White model's rate distributions are not lognormal (in the T-forward-measure) the question arise

- what kind of smile is implied by the model
- how this smile fits a smile observed in the market's vola quotes.

Extra slide

Smiles in the Hull White approach

Comparison of the market quotes of 2006-12-28 for a 5y/5y - swaption with strike -50 BP, at the money, +50 BP and the implied smile of a 2 factor HW-model shows good consistency of the smiles (1 factor model shows correct shape of smile but does not fit as good as in the 2 factor case).



Calibration of Interest Rate Models 1/3

Interest Rate Models are calibrated
- on today‘s yield curve in order to fit this curve today
- on liquidly traded instruments, i.e. at the money caps/floors or swaptions

For the LMM / SMM there are very direct and simple ways of calibrating them to the market‘s implied volatilities, because they imitate the situation in the Black76-Formula which is the vehicle for the market‘s vola-quotes.

Calibration of Interest Rate Models 2/3

The other models must be calibrated by an iterative optimizing procedure:

1. Try a set of real numbers for the model's parameters
2. Price the calibrating instruments in the model
3. Calculate the pricing error model vs. market prices (calculated by Black76 / quoted volas)
4. Minimize the pricing error by repeating steps 1 - 3

For this, different optimizing algorithms can be used. We will come to this later.

It is known that
- Caps / Floor are insensitive to correlations between rates of different maturities but
- Swaptions are sensitive to these correlations.
(this is because caps can be decomposed into independent caplets while there is no such decomposition for swaptions). Therefore we chose swaptions as our calibrating universe.

Calibration of Interest Rate Models 3/3

What subset of the swaption matrix should be used for calibration?

1. Use whole matrix. A problem might be that certain entries are not up to date or referring to non-liquid instruments.
2. Use only liquid instruments. A rule of thumb is to use only the upper left triangle.
3. Choose instruments which are sensitive to the same rates as the instrument to be priced.

We will be using all alternatives in the following.

Numerical methods for pricing 1/3

We will use 2 different methods for actual pricing in the following:

1. Backward Induction in a recombining trinomial tree
2. Forward Monte Carlo simulation

The strength of the tree approach is the pricing of optional elements, because the decision whether or not to exercise these rights early at a node can be easily made upon the values already calculated (specifically the conditional expectation of the discounted payoff is needed to make this decision).

But it is hard to value path-dependent payoffs (like in asian options for example) because the path to a node is not known when calculating this node.

Some path-dependencies can be resolved by a „trick“, though, as we will see later.

Numerical methods for pricing 2/3

The strength of the monte carlo approach is the pricing of path-dependent payoffs, because the path already taken is naturally available in a specific simulation run.

It is more complicated to value optional elements, though still possible:

A technique developed by Longstaff and Schwarz allows to estimate conditional expectations via a linear regression approach. Thereby, optional elements can also be priced.

Numerical methods for pricing 3/3

In practice a good choice seems to be ...

... a tree approach if optional elements has to be priced and no „serious“ path dependency is involved (see later for what is meant by „serious“).

.. a monte carlo approach if „serious“ path dependency is involved. If optional elements are also present, use the Longstaff-Schwarz Approximation for conditional expectations.

Hypotheses

For swaps with payoffs depending only on the marginal distributions of rates with different maturities (or depending only on one fixed maturity rate) the Hull White one factor model should deliver reasonable prices.

For swaps with payoffs depending on the common distribution of rates with different maturities (e.g. a spread between short- and long-term rates) the Hull White one factor model will not deliver reasonable prices.
In this case, the two factor model should be able to give reasonable prices, however.

We will check these hypotheses in the following by pricing concrete swaps and comparing the result both with internal models and 3rd parties' NPVs.

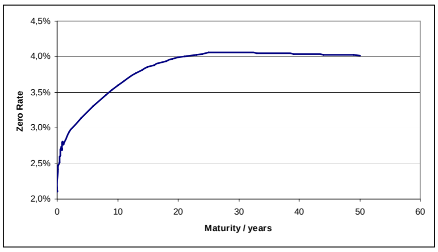
Contents

Market data as of 2005-11-29

ATM Swaption implied volatilities:

Option Maturity \ Swap Maturity	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	25y	30y
3m	19,60%	20,50%	21,00%	20,80%	20,50%	20,00%	19,30%	18,50%	17,80%	17,00%	15,40%	14,60%	14,10%	13,80%
6m	20,80%	21,00%	21,00%	20,60%	20,30%	19,80%	19,20%	18,50%	17,80%	17,10%	15,50%	14,70%	14,20%	13,90%
1y	21,20%	21,00%	20,60%	20,30%	19,90%	19,40%	18,80%	18,20%	17,60%	17,00%	15,80%	15,00%	14,60%	14,20%
2y	20,90%	20,40%	20,00%	19,60%	19,10%	18,60%	18,10%	17,70%	17,20%	16,80%	15,90%	15,30%	14,90%	14,50%
3y	20,40%	19,80%	19,30%	18,80%	18,20%	17,80%	17,40%	17,10%	16,80%	16,50%	15,60%	15,10%	14,70%	14,40%
4y	19,70%	19,10%	18,60%	18,00%	17,50%	17,10%	16,90%	16,60%	16,40%	16,20%	15,30%	14,90%	14,60%	14,30%
5y	19,00%	18,40%	17,90%	17,30%	16,80%	16,50%	16,30%	16,10%	16,00%	15,80%	15,10%	14,70%	14,40%	14,20%
10y	15,60%	15,30%	15,00%	14,80%	14,60%	14,50%	14,40%	14,30%	14,40%	14,30%	13,80%	13,50%	13,30%	13,20%
30y	13,10%	13,10%	13,00%	13,00%	12,90%	12,90%	12,90%	12,90%	12,90%	12,90%	12,50%	12,30%	12,20%	12,10%

Zero yield curve:



Resulting Objective function to minimize

The objective function is defined as the sum of squares of relative pricing errors (model vs. the market (Black76-) prices) of the swaptions chosen for calibrating.

The computation of the market prices is straightforward via the Black76-Formula.

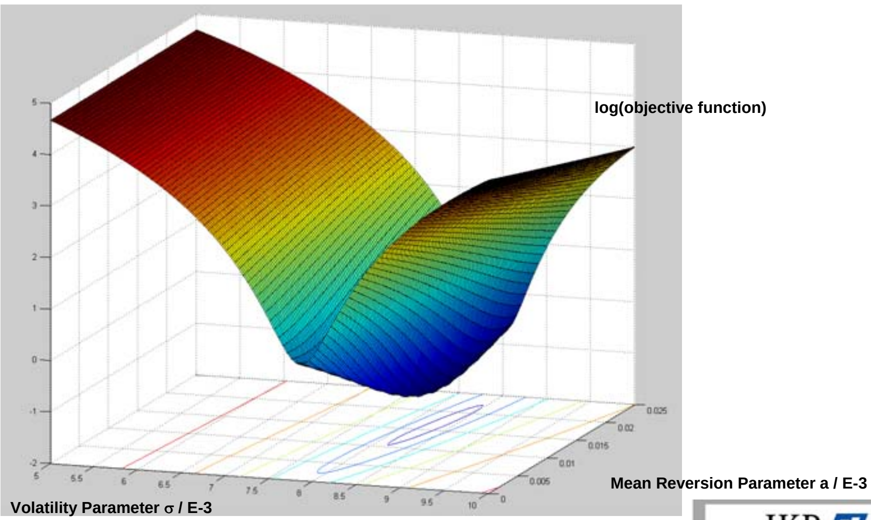
The computation of the model prices is more complicated, but can be analytically done for Hull White 1- and 2-Factor models. Nevertheless, numerical procedures are required to

- compute zeros of 1-dimensional functions with no closed form solution (both 1 and 2 factor)
- compute a 1-dimensional integral with gaussian weight function (only for 2 factor)

It is important to use the Gauss-Hermite procedure to compute the integral because of performance issues. I use 20 sampling points, which seems to give sufficient precision.

Picture of a 1 factor objective function

Market data as of 2005-11-29, full swaption matrix included in calibration.



Optimizing algorithms

The picture for the 1 factor case does not exhibit several local minima. But this seems to be deceptive at least for the 2 factor case.

Indeed the most stable algorithm for calibrating 2 factor models was simulated annealing with a Nelder-Mead procedure as the underlying deterministic optimizer*.

In a simulated annealing algorithm a stochastic component is added to the objective function and „cooled down" to determinism while optimizing. Thereby the algorithm does not get caught in local minima.

Simulated annealing worked much more stable in the 2 factor case than deterministic Nelder Mead, Levenberg-Marquardt and different line search algorithms, which I also tried.

* See „Numerical Recipes in C" for a description

Results from calibration (1 factor case)

Using Market data as of 2005-11-29 and the full swaption matrix for calibration I get the following results in the 1 factor case:

a = 0.016556567550286113
σ = 0.0067951607500858395

Relative pricing errors:

Option Maturity \ Swap Maturity	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	25y	30y
3m	24,76%	13,19%	6,48%	3,84%	2,13%	1,58%	2,29%	3,75%	5,00%	7,31%	7,70%	6,71%	6,11%	5,59%
6m	12,56%	6,62%	2,97%	1,63%	0,08%	-0,34%	-0,08%	0,89%	2,20%	3,95%	4,61%	3,81%	3,32%	2,88%
1y	4,76%	2,44%	1,04%	-0,42%	-1,34%	-1,57%	-1,21%	-0,62%	0,31%	1,65%	0,39%	-0,15%	-1,15%	-0,82%
2y	0,76%	-0,18%	-1,02%	-1,85%	-2,00%	-2,08%	-1,97%	-2,10%	-1,33%	-1,08%	-2,91%	-4,15%	-4,76%	-4,28%
3y	-2,44%	-2,05%	-2,29%	-2,33%	-1,81%	-2,15%	-2,19%	-2,45%	-2,72%	-2,51%	-3,53%	-4,75%	-4,95%	-4,90%
4y	-3,08%	-2,90%	-2,92%	-2,39%	-2,13%	-2,09%	-2,83%	-3,05%	-3,34%	-3,68%	-3,94%	-5,17%	-5,62%	-5,38%
5y	-4,20%	-3,59%	-3,54%	-2,68%	-1,98%	-2,02%	-2,74%	-2,92%	-3,78%	-4,10%	-4,83%	-5,46%	-5,53%	-5,75%
10y	-1,14%	-1,51%	-0,31%	-0,32%	-0,56%	-0,34%	-0,40%	-0,63%	-2,33%	-2,77%	-2,01%	-1,60%	-1,61%	-2,52%
30y	9,37%	8,49%	8,39%	7,51%	7,43%	6,57%	5,72%	4,88%	4,06%	3,26%	2,98%	1,30%	-1,35%	-3,56%

The average relative pricing error is 3.38%.

Results from calibration (2 factor case)

Using Market data as of 2005-11-29 and the full swaption matrix for calibration I get the following results in the 2 factor case:

$a = 2.5077133266699736$
$b = 0.020476892702677017$
$\sigma = 0.006653290973920811$
$\eta = 0.007129960712319258$
$\rho = -0.9460339775586781$

Relative pricing errors:

Option Maturity \ Swap Maturity	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y	15y	20y	25y	30y
3m	-0,44%	2,52%	0,83%	0,42%	-0,08%	0,09%	1,25%	2,99%	4,43%	6,86%	7,31%	6,04%	5,07%	4,19%
6m	-3,73%	-0,08%	-0,28%	-0,05%	-0,75%	-0,67%	-0,12%	1,04%	2,46%	4,27%	4,80%	3,62%	2,73%	1,89%
1y	-3,22%	-0,15%	0,42%	-0,15%	-0,61%	-0,60%	-0,11%	0,54%	1,49%	2,83%	1,20%	0,18%	-1,27%	-1,35%
2y	-1,48%	0,21%	0,30%	-0,16%	-0,14%	-0,17%	-0,07%	-0,25%	0,47%	0,63%	-1,75%	-3,52%	-4,63%	-4,58%
3y	-2,73%	-0,69%	-0,37%	-0,21%	0,37%	0,01%	-0,10%	-0,44%	-0,83%	-0,72%	-2,34%	-4,12%	-4,82%	-5,21%
4y	-2,53%	-1,16%	-0,80%	-0,14%	0,12%	0,10%	-0,74%	-1,07%	-1,48%	-1,94%	-2,81%	-4,63%	-5,58%	-5,79%
5y	-3,24%	-1,69%	-1,38%	-0,45%	0,24%	0,12%	-0,72%	-1,02%	-2,02%	-2,47%	-3,82%	-5,04%	-5,61%	-6,28%
10y	0,18%	0,22%	1,51%	1,44%	1,10%	1,20%	1,01%	0,63%	-1,23%	-1,81%	-1,72%	-1,91%	-2,45%	-3,80%
30y	8,41%	7,64%	7,47%	6,47%	6,24%	5,23%	4,23%	3,25%	2,29%	1,34%	0,36%	-1,88%	-4,94%	-7,47%

The average relative pricing error is 2.18%.

Summary of calibration

Both models show a good replication of the swaption market prices „on the whole“.

The 2 factor model, however, adapts better to the swaption matrix than the 1 factor model, especially in the upper left corner of the swaption matrix.

This is due to the ability of the two factor model to produce non-parallel-shifts to the yield curve better than the one factor model (correlation of 1 between different rates!).

The calibration was done for several dates in order to price the swaps on different points in time. There were no special observations beyond the ones above.

Also, the calibration was done for shifted yield curves and swaption matrices in order to calculate sensitivities (BPV and Vega).

Contents

Multi callable accreting swaps - description

Example (Original nominal is substituted by an artificial value):

Nominal = 1.000.000 EUR

Start Date = YYYY-01-15

End Date = (YYYY+7)-11-08

Bank receives yearly fixed rate cupons of 5.19%

Bank pays half-yearly 6m-Euribor

The nominal is increased by the fixed rate‘s compound factor each year

The payer of the fixed rate has the right to call the swap yearly on Nov, 8th starting in YYYY+2.

This is a Level 3 – swap, because the bermudan style option can not be valued analytically.

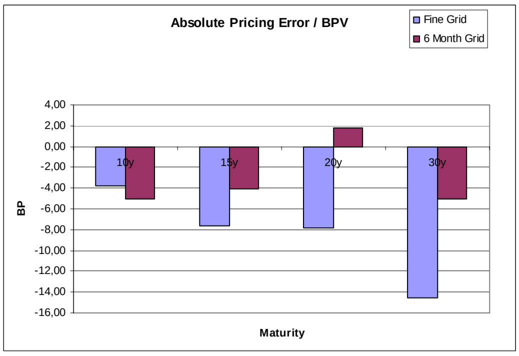
I use the tree approach to price this swap.

I also price swaps of the same type, but with longer maturities of 15y, 20y and 30y.

Multi callable accreting swaps - results

Valuation Date: 29.11.XXXX - Calibration was done on whole swaption matrix - Tree uses 8 grid points per month

Deal		Internal Model			Own 1 Factor Valuation			Errors			2 Factor	6 Month Grid	Error
Deal No.	Maturity	NPV	BPV	Vega	NPV	BPV	Vega	relative	abs / BPV	abs / Vega	NPV	NPV	abs / BPV
1	10y	47.970	-258	-557	48.954	-242	-412	2%	-3,81	-1,77	48.906	49.261	-5,00
2	15y	90.094	-499	-2.104	93.891	-555	-1.930	4%	-7,60	-1,80	93.030	92.122	-4,06
3	20y	-31.197	-1.165	-7.019	-22.135	-754	-6.683	29%	-7,78	-1,29	-23.676	-33.233	1,75
4	30y	9.108	-838	-7.838	21.314	-631	-5.072	134%	-14,57	-1,56	20.237	13.316	-5,02



1- and 2-Factor models produce very similar valuations*.

The pricing error increases systematically with growing maturity.

A coarser grid shows better consistency.

It should be checked whether this may be due to a coarse grid in the internal calculations or some other problem (open issue).

* Similar NPVs are produced when calibrating on instruments with option length + swap length = 30y

Volatility swaps - description

Example (Original nominal is substituted by an artificial value):

Nominal = 1.000.000 EUR

Start Date = YYYY-01-15

End Date = (YYYY+10)-01-15

Bank pays one fixed rate cupon of 3.45% for the first year

Bank pays max(6.65% - 5 x Volaindex ; 2.00%) yearly for years 2 – 9

Volaindex = Arithmetic Mean of absolute differences between the 20y forward swap rate starting on (YYYY+10)-01-15 fixed on 8 fixing dates before the start of each period. The fixing dates are quarterly, so they cover the last 2 years before the start of the period. The difference is taken 4 times between the rates fixed with exactly one year between them.

Bank receives 6m-Euribor.

This is a Level 3 – swap, because of the structured cupons depending on the volatility of a forward rate.
I use the monte carlo approach to price this swap because of the path dependency. This dependency could be resolved, but this would be very cumbersome.
(This type of swap can be approximated by a portfolio of level 2 – swaps, though, as we will see later.)

Volatility swaps - results

Valuation Date: 29.11.XXXX - Calibration was done on whole swaption matrix - Monte Carlo uses 2.500 simulation runs
A 99%-confidence intervall is calculated for the MC-estimator and given below the NPV

Deal		Internal Model			Own 1 F	Own 2 F	Errors (1 Factor)		
Deal No.	Maturity	NPV	BPV	Vega	NPV	NPV	relative	abs / BPV	abs / Vega
5	10y	-74.884	950	7.979	-74.937	-76.166	0%	-0,06	-0,01
					+/- 1.670	+/- 1.660			

Both 1 and 2 Factor models are highly consistent with the internal calculations.

The internal models NPV, however, is calculated by an analytical approximation which uses a portfolio of strangles and straddles which approximately replicates the payoff of the swap and which can be priced by ordinary Black76-calculations.

TARN - description

Example (Original nominal is substituted by an artificial value):

Nominal = 1.000.000 EUR

Start Date = YYYY-08-22

End Date = (YYYY+7)-08-22

Bank pays half-yearly max(8.1% – 2 x 6m-Euribor ; 0) (fixing is in arrears)

Bank receives half-yearly min(6m-Euribor ; 5.0%) (fixing is in arrears)

Swaps terminates early if a the sum of already paid cupons (by Bank) exceeds 8.5% of the nominal.

This is a Level 3 – swap, because of the early stopping condition.

I use the monte carlo approach to price this swap because of the path dependency.

TARN - results

Valuation Date: 31.12.XXXX - Calibration was done on whole swaption matrix - Monte Carlo uses 10.000 simulation runs
A 99%-confidence intervall is calculated for the MC-estimator and given below the NPV

Deal		Internal Model			Own 1 F	Own 2 F	Errors (1 Factor)		
Deal No.	Maturity	NPV	BPV	Vega	NPV	NPV	relative	abs / BPV	abs / Vega
6	7y	81.541	872	-1.665	103.405	104.843	27%	25,08	-13,13
					+/- 2.670	+/- 2.630			
		in advance fixing (both legs)			80.116	82.405	2%	-1,63	0,86
					+/- 2.370	+/- 2.400			

The 1- and 2-factor models produce similar NPVs*, both, however, significantly deviating from the internal calculation.

If I assume „in advance fixing“ instead of „in arrears fixing“, the NPVs are consistent.

It should be checked which assumption about the fixing is made in the internal model. Furthermore it should be checked what fixing mode is agreed on with the counterparty (open issue).

*Similar NPVs are produced when calibrating on instruments with option length + swap length = 7y

Bandbreiten Swap - description

Example (Original nominal is substituted by an artificial value):

Nominal = 1.000.000 EUR

Start Date = YYYY-03-30

End Date = (YYYY+6)-09-30

Bank pays quarterly 3m-Euribor (fixing is in advance)

Bank receives quarterly 3.3%, 3m-Euribor or 4.95% depending on whether the 3m-Euribor is below 2%, between 2% and 4.95% or above 4.95%. (fixing is in arrears)

This is a Level 3 – swap, because of the special shape of the payoff profile.

A monte carlo as well as a tree approach can be taken to price this type of swap and I use both of them.

In the results only the monte carlo results are shown, but the tree results are consistent with them.

Bandbreiten Swap - results

Valuation Date: 31.12.XXXX - Calibration was done on whole swaption matrix - Monte Carlo uses 4000 simulation runs
A 99%-confidence intervall is calculated for the MC-estimator and given below the NPV

Deal		IM	3rd Party	1 Factor	2 Factor	Error IM	Error 3rd P
Deal No.	Maturity	NPV	NPV	NPV	NPV	relative	relative
7	6,5y	10.226	14.203	17.747	15.885	74%	25%
				+/- 1.400	+/- 1.430		
	in advance fixing			13.961	13.234	37%	2%
				+/- 1.320	+/- 1.430		

The 1- and 2-factor models produce similar NPVs*, both however significantly deviating from the internal calculation and also from the 3rd party's calculation.

If I again assume „in advance fixing", I get nearly the same NPV as the 3rd party, though.

It should be checked what fixing mode is agreed on with the counterparty and what assumption Is used for the internal calculation. (open issue)

The internal model uses an analytical approximation for the valuation. It should be checked, if it is really superior (w.r.t. 3rd parties' valuations) to the 1 factor-valuations. (open issue)

*Similar NPVs are produced when calibrating on instruments with option length + swap length = 7y

Contents

Bandbreiten Spread Swap - description

Example (Original nominal is substituted by an artificial value):

The first component is a Bandbreiten Swap with Nominal = 1.000.000 EUR, Start Date = YYYY-05-06, End Date = (YYYY+10)-02-06, Barriers 1,85% and 3,70% and interest rates of 3,30%, 3m-Euribor-70BP, 3,70% to be received by Bank depending on where the 3m-Euribor lies w.r.t. the barriers.

The second component is defined as follows:
Bank receives 2% if the difference between the 30y and the 2y swap rate is below 1,25%
Bank pays 2% if the difference between the 30y and the 2y swap rate is above 2,00%

This is a Level 3 – swap, because of the special shape of the payoff profile.
A monte carlo as well as a tree approach can be taken to price this type of swap and I use both of them.
In the results only the monte carlo results are shown, but the tree results are consistent with them.

Bandbreiten Spread Swap - results

Valuation Date: 31.12.XXXX - Calibration was done on whole swaption matrix - Monte Carlo uses 500 simulation runs
A 99%-confidence intervall is calculated for the MC-estimator and given below the NPV

Deal		3rd Party	Internal Model			1 Factor	2 Factor	Errors (1 Factor)		
Deal No.	Maturity	NPV	NPV	BPV	Vega	NPV	NPV	relative	abs / BPV	abs / Vega
8	9,75y	80.343	70.180	-297	-313	121.675	118.826	73%	-173,56	-164,73
						+/- 3.210	+/- 3.500			
					Spread component's NPV	160.311	159.099			
						+/- 700	+/- 650			

The 1- and 2-factor models produce similar NPVs*, both, however, significantly deviating from the internal calculation and also from the 3rd party's calculation.

Obviously, the spread component has a very small variance in the monte carlo simulation. Almost all outcomes must be the „spread below 1,25% => Bank receives 2%" – case.

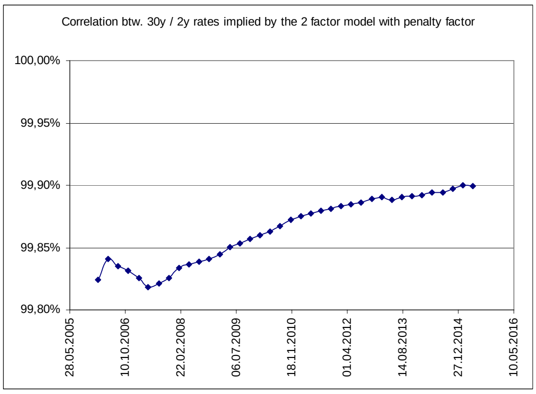
For the 1-factor model this is not surprising, because here a correlation of +1.0 between 30y and 2y rates is implied by the model, so the spread between them will (almost) be constant (and obviously below 1,25%).

For the 2-factor model this is not necessarily the case, so let's have a closer look at it.

*Similar NPVs are produced when calibrating on instruments with option length + swap length = 10y

30y / 2y correlation implied by 2 factor model

The correlation between the 30y and 2y swap rate which is implied by the 2 factor model can easily be measured in the monte carlo simulation. The result is as follows:



The 2 factor model also implies correlations which are near +1.0.

What now?

Obviously the hull white 1- and 2-factor models are not suitable to price the spread component of the swap because of the high implied correlation between 2y and 30y rates.

In the internal model this problem is solved by an analytical approximation approach which produced the NPV shown earlier which is in acceptable consistency with the 3rd party's NPV.

I tried 2 „self-made“ fixes, one „ad hoc“ and one which seems to be more sophisticated:

- Manual decorrelation in the 1 factor model
- Calibration of the 2 factor model with a penalty factor for high correlations

Spread pricing by „Decorrelation“

Idea: Use the 1 factor model, but „decorrelate“ the rates to a given correlation.

Specifically:

1. Run n steps in the monte carlo simulation to determine the marginal distributions of the 2y and 30y rates for each time point which is needed for pricing.
2. Run m steps in the monte carlo simulation to price the swap. Use the marginal distributions from step 1 and join them by a gaussian copula with externally given correlation. Sample from this 2-dimensional distribution to price the spread component.

Which correlation to use? I tried to estimate the correlation – roughly – from historical data (15 data points 01.10.05 to 10.12.05), which lead to apprx. 85%

Correlation	1F / Decorr NPV	3rd Party NPV	IM NPV
85%	97.306	80.343	70.180
	+/- 1.800		

The result seems to high, but not too bad with regard to the relatively rough correlation estimation.

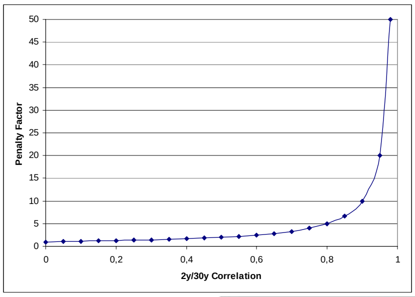
Spread pricing by G2 with penalty factor

Idea: Try to enforce lower correlations in the calibration of the 2 factor model by introducing a penalty factor which leverages the objective function depending on the implied correlation between 2y and 30y rates.

Specifically I modified the objective function to

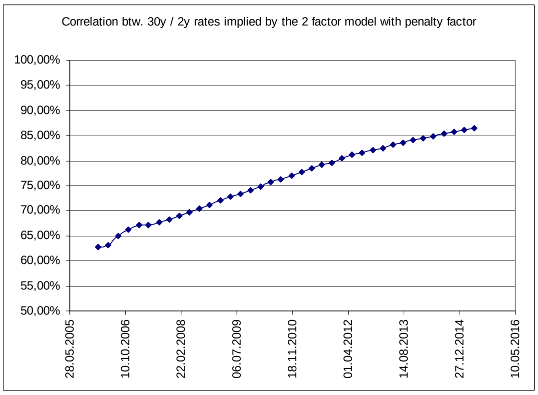
Original objective function x Penalty factor

I chose the penalty factor as 1 / (1-CORR) where CORR is the correlation of 2y and 30y rates measured at „today+10y“.



Calibration results for the modified G2

The average swaption pricing error is 1.12% in sample and 3.52% out of sample in the (bilinearly interpolated) {1y, ... , 10y} x {1y, ... , 10y} – swaption matrix. The correlation structure now looks as follows:



Pricing results for the modified G2

Modified G2	3rd Party	IM
NPV	NPV	NPV
77.985	80.343	70.180

+/- 2.700

The result is quite consistent with both the internal calculation and the 3rd party's NPV, in fact even nearer to the 3rd party's NPV. This may be due to the analytical approximation procedure which is used in the internal calculation.

Spread pricing by a BGM approach

Although the two „fixes“ to the G2 model presented lead to good results, the usage of a BGM model seems to be the „natural“ choice to price spread products because of the greater flexibility in modelling the correlation structure between rates.

As stated earlier, however, low-factor versions such as a 1 factor BGM with result in two low correlation between the 2y and 30y rate in our example and thus to a too low price (in contrast to the Hull White model where a too high price is produced because of the too high correlation).

So a careful specification of the correlation structure is needed and the correlation needed to price the spread should be checked explicitly in the resulting model.

Summary of results 1/2

For swaps with payoffs depending only on the marginal distributions of rates with different maturities (or depending only on one fixed maturity rate) the Hull White one factor model is a reasonable choice for pricing.

For swaps with payoffs depending on the common distribution of rates with different maturities (e.g. a spread between short- and long-term rates) the Hull White one factor model does not deliver reasonable prices.
In Principle, the two factor model is able to give reasonable prices. In my calibrations (without penalty factor), however, a very high correlation was always implied by the model, so it „behaves“ similar to a one factor model.

As a fix for this model class we introduced
- „decorrelation“ of the one factor model and
- calibration of the two factor model while penalising high correlations

Summary of results 2/2

When using a BGM approach, the resulting correlation structure must carefully be checked before pricing an instrument.
Low factor representations (e.g. 1 factor BGM models) will not give reasonable prices because of implausible correlations between both adjacent and distant rates.

Furthermore some very concrete issues arose which will have to be checked:

- When pricing MC Accreting Swaps a coarse grid showed better consistency with the internal calculation than a fine grid. What is the reason for this?

- When pricing TARNs and Bandbreiten Swaps the assumption of „in advance fixing" showed better consistency with internal calculation than „in arrears fixing" (which is the fixing mode in the trade system, should be checked against to the original contract)

Contents

Discussion

Thank you for your attention.

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Extra slide

Eliminating some path dependencies in trees

Some „well behaved“ path dependencies can be eliminated when going backward in time in a valuation tree. The idea is as follows:

If a payoff at time t depends on the yield curve at time s < t, take a preliminary payoff of e.g. 1 at time t and adjust it to the right value at time s.

When doing this, it will be generally necessary to separate payoffs which are made at different time points, adjust them separately and not to consolidate them until they are adjusted.

Therefore I introduced „slots“ in the tree. The payoff can be made in a specified slot, the slots can be adjusted separately and consolidated at defined points in time.

Extra slide

Eliminating some path dependencies in trees

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Thus, some path dependencies can easily be eliminated such as

- in advance fixed payments (this is a path dependency also!)
- stochastic accreting and amortizing structures

and some more. It may be quite cumbersome to manage the slots, their adjustments and consolidations, though.

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Extra slide

Software design: Payoff profile for a tree

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The payoff of a product is described by a class defining the methods of an abstract Class „AssetValue“ which essentially looks like (in a java-styled pseudo code notation):

```
abstract class treeAssetValue {
  double getPayment(int adjustmentSlot, Date paymentDate, YieldTermStructure yts)
  boolean isEarlyExercisePossible(Date exerciseDate)
  double getEarlyExerciseValue(Date earlyExersiceDate, YieldTermStructure yts)
  double getAdjustmentFactor(int adjustmentSlot, Date adjustmentDate, YieldTermStructure yts)
}
```

Input to the methods are the date of a possible payment and the yield term structure at this date (which is abstract itself because dependent on the model used).
Output of the methods are the resulting payments (eventually for each slot), early exercise possibility and value and eventually an adjustment factor to be applied to a certain slot to eliminate path dependency.

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Extra slide

Software design: Payoff profile for mc

65

The payoff of a product which is to be priced in a mc approach is a bit different:

```
abstract class mcAssetValue  {
 double getPayment(Date d, Map yieldTermStructures)
}
```

Now, the method returning a payment gets all yield term structures prior to the present date d as they were simulated in a specific run of the simulation.

(The absence of early exercise – methods is a result of my not implementing a Longstaff-Schwarz approach yet in the simulation engine.)

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Extra slide

Software design: Abstract pricing engine

66

The abstract pricing engine is described as follows:

```
public interface pricingEngine {
  double getTreeAssetValue(AssetValue av);
  MeanVarianceCollector getMCAssetValue(MCAssetValue av, int simulations);
}
```

Given an arbitrary asset value description, the valuation tree has to return the NPV of this payoff profile. The first method returns the NPV calculated in a tree approach, the second the one calculated in a mc approach.

In the mc approach the number of simulations to be run has to be specified. Furthermore, not only the expectation of the NPV is returned but also the variance from which a confidence interval for the simulation results can easily be derived.

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Extra slide

Software design: Put it together

Now, for each product an implementation of the abstract asset value classes have to be provided.

For each model which shall be used (yet only the hull white 1 factor and 2 factor models) an implementation of the abstract pricing engine class has to be provided.

Using the abstract descriptions of products' pay off profiles and pricing engines, both new products and new models can easily be added with no changes necessary in the code for existing products and models.

Extra slide

Software design: Flaws (and remedy)

It would be even a better design, if a single interface would suffice to describe the pay off. Then, the pricing engine, too, would have to implement only a single method and it would be its own responsibility to „choose" a certain numerical method.

It would be no serious effort to refactor the design it order to achieve this.